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The Portfolio Optimization Project

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The Portfolio Optimization Project

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Portfolio Optimization Project

Abstract:

This project has three parts. The first part is to use the efficient frontier and find the tangency portfolio to form our optimal portfolio. We built our portfolio using the Interactive Brokers software[1] and rebalanced every week for 4 holding periods to see the relationship between our projected returns and actual market returns. In the second part we considered the Capital Asset Pricing Model (CAPM) and ran linear regressions on the stocks we chose in the first part of the project. This process is based on our idea of finding the systematic risk in each stock to improve our stock choosing ability. In the last part we introduce the concept of factor models and add more factors into our original CAPM model. Via a back-testing method, we test the reasonability of our factors and give advice to further improve our portfolio optimization project.

[1] Provided by Interactive Brokers Group, Inc. (IB), which is an online discount brokerage firm in the United States.

Chapter 1: PORTFOLIO OPTIMIZATION

1. INTRODUCTION AND OBJECTIVE:

The concept of the efficient frontier, which is an essential component in Modern Portfolio Theory, was introduced by Harry Markowitz[2]. We have a group of assets in our investment basket. For every combination of assets (each asset can have a positive or negative weight while the sum of the weights is 1) there should be exactly one expected return and level of risk (usually proxied by the standard deviation of the portfolio's return). For any fixed expected return there must be a minimum portfolio risk that can be reached by exactly one specific asset combination. This specific portfolio is said to be lying on the parabola-shaped curve called the "efficient frontier" like Figure 1 shows us below. Portfolios located on the efficient frontier are called efficient portfolios or mean-variance efficient portfolios. An efficient portfolio has the highest expected return under a specific risk level and has the lowest risk under a specific expected return. No combination of assets would have a corresponding pair of expected return and standard deviation located to the left of the frontier.

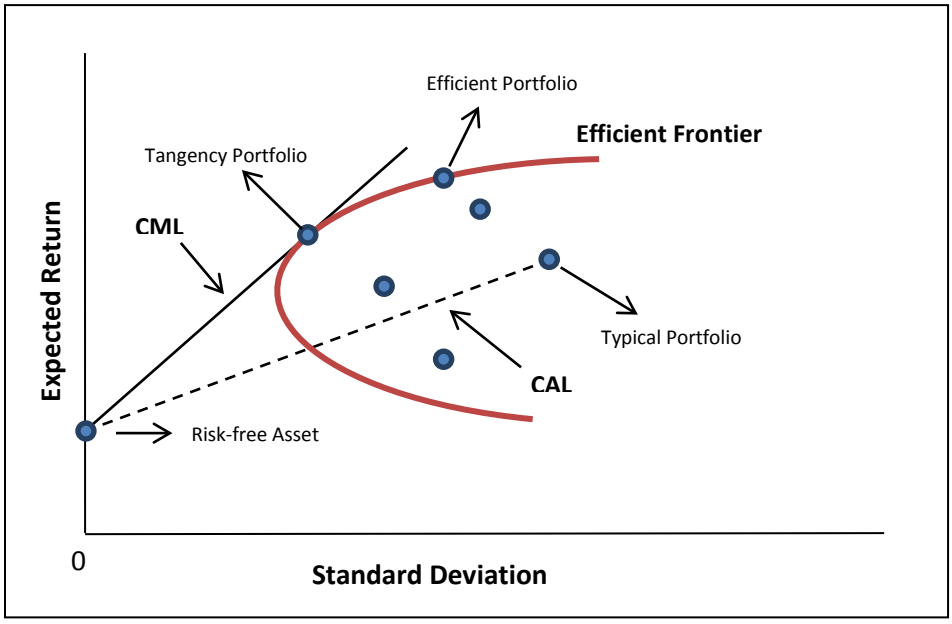


Figure 1: Efficient Frontier and CML

Once we have the efficient frontier, we may desire to combine those risky portfolios with a risk-free asset to optimize our efficient frontier. Since the risk (standard deviation) of a risk-free asset is zero, the combination of the risk-free asset and a typical portfolio lies on the dashed line exhibited in the graph, and we can call the line the capital allocation line (CAL). Among all the CAL's, we can derive the capital market line (CML) by drawing a tangent line from the intercept point (called the tangency portfolio) on the efficient frontier to the point where the expected return equals the risk-free rate of return. The CML is

[2] Markowitz, H.M. (March 1952). "Portfolio Selection". *The Journal of Finance* 7 (1): 77–91

considered to be superior to the efficient frontier since it takes into account the inclusion of a risk-free asset in the portfolio[3]. The CML can be treated as our advanced efficient frontier.

Our theory and methods here are based on several assumptions[4]:

- Every investor acts as an efficient investor. Rational investors will only form portfolios located on the efficient frontier.
- All investors have the same probability distribution for future rates of return.
- All investors can lend or borrow money at the risk-free rate of return.
- There are no transaction costs or taxes associated with the purchasing or selling of assets.
- All investments have a similar time-line across which they are measured. This infers that all investments are modeled within a relative time-space continuum.

In this project, our first goal is to use the efficient frontier to form our optimal tangency portfolio. Then we track our portfolio using an Interactive Brokers Account[5] with weekly rebalancing.

2. STOCK AND FACTOR CHOICE:

To form our investment pool, we chose 15 stocks trading on the New York Stock Exchange (NYSE). Those stocks were chosen from different sectors for the purpose of diversification. They also have the large market capitalizations to insure marketability and stability. Some of the firms are headquartered outside the U.S. (e.g. BP, NOK), which makes our international premium factor more meaningful.

Table 1: Symbols, Sectors and Market Capitalizations of the 15 stocks

Name	Symbol	Sector	Market Cap
Bank of America Corporation	BAC	Financial	51.17B
Citigroup, Inc.	C	Financial	82.36B
JP Morgan Chase & Co.	JPM	Financial	122.78B
Lloyds Banking Group Plc Americ	LYG	Financial	26.67B
Wells Fargo & Company	WFC	Financial	137.48B
EMC Corporation	EMC	Technology	47.45B
Corning Incorporated	GLW	Technology	20.78B
Nokia Corporation Sponsored Ame	NOK	Technology	20.85B
BP p.l.c.	BP	Basic Materials	136.78B
Freeport-McMoRan Copper & Gold	FCX	Basic Materials	37.25B
Las Vegas Sands Corp.	LVS	Services	33.58B
Time Warner Inc. New	TWX	Services	34.44B
Ford Motor Company	F	Consumer Goods	41.42B
General Electric Company	GE	Industrial Goods	169.87B
Pfizer, Inc.	PFE	Healthcare	152.89B

[3] http://www.investopedia.com/ Investopedia
[4] Refer to 'David Ruppert, Statistics and Finance: An Introduction'
[5] Using the electronic trading platform called Trader Workstation (abbreviated TWS), provided by IB.

To choose the risk-free rate of return, we first considered the three-month U.S. Treasury bill on Oct 1st, 2011, whose annual interest rate was 0.0008. Although a three-month Treasury bill was commonly used as the risk-free rate for decades, recent experience indicates that this convention should be doubted because of the strong intervention of the Federal Reserve. For example in November 2010, the Fed announced a second round of quantitative easing, or "QE2", buying $600 billion of Treasury securities by the end of the second quarter of 2011.[6] [7] We decided to use the Moody's Seasoned Aaa Corporate Bond Yield on Nov 4th, 2011 (the date we form our IB portfolio), which was 3.88%. Since our portfolio holding period is one week, we generate the weekly returns of those 15 stocks to form our database[8].We divided our annual risk-free interest rate by 52 (number of weeks in a year) to get our program's risk-free input.

If the data range we choose is too long (like 20 years), the financial conditions may change and the stocks' intrinsic return variances and covariances may change. If the time range is too short (like 3 weeks), the uncertainty of the price volatility maybe extremely high and the covariance matrix we used to form our portfolio maybe questionable. Thus on every rebalancing day (including the starting date of our portfolio), we looked back 6 months as the range for our database.

3. INITIAL PORTFOLIO FORMATION:

In this section, we will illustrate our steps for forming our portfolio. We will show the program results we generated, the difficulties we encountered during the mark-to-market trading using the Interactive Brokers software, and our modified processes.

1) Nov 2nd, 2011 (Wednesday)
   Initially we set no bounds on our stock positions. That means we can short or long a stock with a very large weight (like having a weight of negative 150.28% in the stock BAC).
   With this assumption, we ran our program and generated Figure 2. The portfolio weights are shown in Figure 3 (data generated after the market closed):

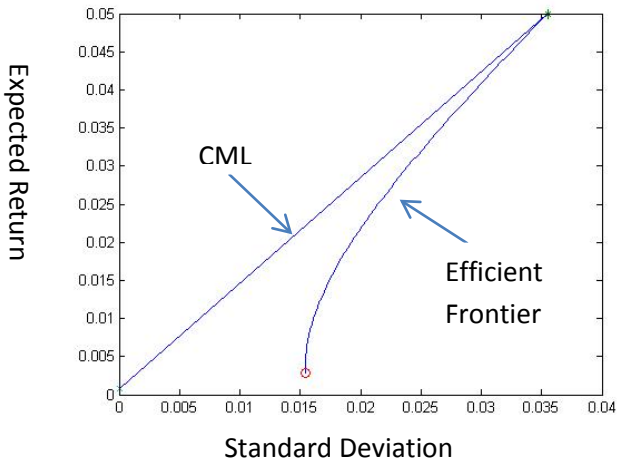


Figure 2: Efficient Frontier and the tangency portfolio calculated from the data up to Nov 2nd, 2011

[6] Censky, Annalyn (3 Nov 2010). "QE2: Fed pulls the trigger".
[7] http://useconomy.about.com/od/glossary/g/Quantitative-Easing.htm US Economy
[8] http://finance.yahoo.com Yahoo Finance

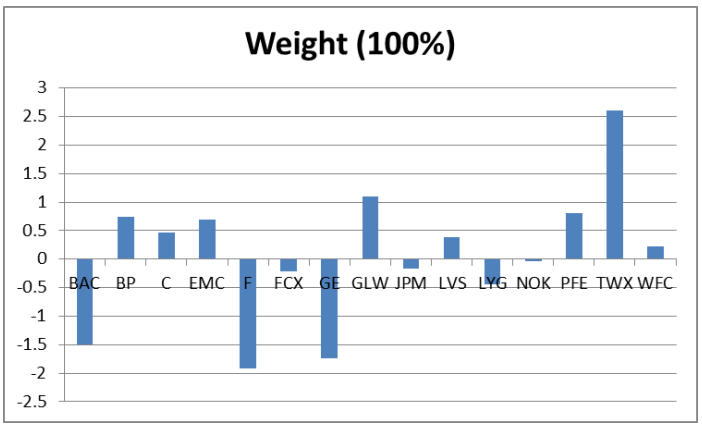


Figure 3: Portfolio Weights

2) Nov 3rd, 2011 (Thursday)
We formed our portfolio according to the weights obtained on the prior night, with a total initial capital of $1,000,000 and a target portfolio value of $500,000. Unfortunately at the end of the last exchange hour, the Interactive Brokers system automatically sold and bought back some of our stocks due to a margin violation. That sudden shock destroyed our portfolio weights and forced us to rebalance them the next day.
3) Nov 4th, 2011 (Friday)
To avoid a margin call, we made two adjustments in our project:
    - Set a lower bound of -25% on the short position of each stock. That means we can no longer short sell a stock with the single asset value more than 25% of total portfolio value.
    - We reset our account to $100,000,000 to ensure enough margin power and try to avoid such uncomfortable events.

    Then we regenerated our portfolio using the adjusted program and rebalanced it in our account. Table 2 shows the rebalanced states and the average purchase prices here are the results of several transactions from 11/3 to 11/4.

Table 2: Rebalance value up to Nov 4th, 2011

11/4/2011	Objective Weights($)	Average Purchasing Price($)	Position	Book Value($)
BAC	-125000	6.61	-19084	-126145
BP	301621.2124	43.24	6888	297837.1
C	-120871.431	30.12	-3972	-119637
EMC	-86179.9331	24.54	-3515	-86258.1
F	-125000	11.16	-11072	-123564
FCX	-79944.8145	40.53	-1972	-79925.2
GE	-125000	16.4	-7622	-125001
GLW	-125000	14.36	-8440	-121198
JPM	126451.9759	33.61	3696	124222.6
LVS	118397.5965	47.55	2466	117258.3
LYG	-117699.615	1.8	-64670	-116406
NOK	3612.963041	6.75	530	3577.5
PFE	458028.2672	19.64	23121	454096.4
TWX	339868.4699	34.18	9733	332673.9
WFC	56715.30813	25.38	2228	56546.64
Portfolio	**500000**			**488078.6**

At this point we successfully formed our portfolio and the scheduled rebalancing dates were 11/14, 11/21 and 11/28. We expected to close all the positions on 12/5.

4. REBALANCING

Here we detail our rebalancing processes. Each period starts at the rebalancing date and ends on the next Friday.

1) Nov 4th (Friday) to Nov 11th (Friday)

The portfolio and market performance for this holding period are shown in Table 3.

Table 3: The Portfolio and Market Performance from Nov 4th, 2011 to Nov 11th, 2011

11/4/2011	Objective Weights($)	Average Purchasing Price($)	Position	Book Value($)	11/11/2011	Close Price($)	Market Value($)
BAC	-125000	6.61	-19084	-126145		6.21	-118512
BP	301621.2124	43.24	6888	297837.1		44.01	303140.9
C	-120871.431	30.12	-3972	-119637		29.33	-116499
EMC	-86179.9331	24.54	-3515	-86258.1		24.64	-86609.6
F	-125000	11.16	-11072	-123564		11.14	-123342
FCX	-79944.8145	40.53	-1972	-79925.2		39.86	-78603.9
GE	-125000	16.4	-7622	-125001		16.3	-124239
GLW	-125000	14.36	-8440	-121198		15.19	-128204
JPM	126451.9759	33.61	3696	124222.6		33.28	123002.9
LVS	118397.5965	47.55	2466	117258.3		46.37	114348.4
LYG	-117699.615	1.8	-64670	-116406		1.83	-118346
NOK	3612.963041	6.75	530	3577.5		6.75	3577.5
PFE	458028.2672	19.64	23121	454096.4		19.99	462188.8
TWX	339868.4699	34.18	9733	332673.9		35.21	342698.9
WFC	56715.30813	25.38	2228	56546.64		25.65	57148.2
Portfolio	**500000**			**488078.6**			**511751.3**

Further tracking our stocks and portfolio returns along with the projected returns, we formed Table 4. This week's projected return is

$$\frac{497994.13}{488078.64} - 1 = 2.032\%.$$

Although we assume no transaction fees and taxes, we consider the dividends received since the distribution of the dividend will force the stock price downward. During this week we received \$0.20 per share of dividend from PFE (long 23121 shares in our portfolio) on Nov 8th. So our actual return for this period is

$$\frac{511751.3+0.2\times 23121}{488078.64} - 1 = 5.798\%.$$

Table 4: Projected Return VS Actual Return

Period 1	Projected Return($)	Initial Market Value($)	Projected Value($)	Market Value($)
BAC	-0.01971097	-126145.24	-123658.794	-118511.6
BP	0.002030213	297837.12	298441.7927	303140.88
C	-0.01143554	-119636.64	-118268.53	-116498.8
EMC	-0.00164223	-86258.1	-86116.4442	-86609.6
F	-0.00952748	-123563.52	-122386.271	-123342.1
FCX	-0.00493985	-79925.16	-79530.3421	-78603.92
GE	-0.00539017	-125000.8	-124327.024	-124238.6
GLW	-0.01171343	-121198.4	-119778.751	-128203.6
JPM	-0.00853226	124222.56	123162.6614	123002.88
LVS	0.006702519	117258.3	118044.226	114348.42
LYG	-0.01915944	-116406	-114175.727	-118346.1
NOK	-0.00531281	3577.5	3558.493429	3577.5
PFE	-6.46E-05	454096.44	454067.0928	462188.79
TWX	-0.0004054	332673.94	332539.0755	342698.93
WFC	-0.00219237	56546.64	56422.66889	57148.2
Portfolio		**488078.64**	**497994.1271**	**511751.3**

2) Nov 14th (Monday) to Nov 18th (Friday)

This week we used the last Friday's market value \$511,751.3 as our new total portfolio capital and our program generated the following objective weights after adding the new week's data and looking back 6 months. Then we marked to market our new stock weights and thus rebalanced our portfolio as Table 5 shows.

Table 5: The Portfolio and Market Performance from Nov 14th, 2011 to Nov 18th, 2011

11/14/2011	Objective Weights($)	Average Purchasing Price($)	Price When Adjusting($)	Close Price($)	Position	Book Value($)	Weights When Adjusting($)	Market Value($)	11/18/2011	Close Price($)	Market Value($)
BAC	-127937.825	6.56	6.07	6.1	-21077	-138265	-127937.39	-128570		5.78	-121825
BP	257165.2326	43.24	43.76	43.71	5877	254121.5	257177.52	256883.7		42.48	249655
C	-127937.825	29.93	28.54	28.47	-4483	-134176	-127944.82	-127631		26.28	-117813
EMC	-66354.42344	24.58	24.399	24.47	-2713	-66685.5	-66194.487	-66387.1		23.07	-62588.9
F	-127937.825	11.16	11.09	11.01	-11536	-128742	-127934.24	-127011		10.1	-116514
FCX	-127937.825	40.2	39.68	39.8	-3224	-129605	-127928.32	-128315		36.94	-119095
GE	-127937.825	16.39	16.27	16.11	-7863	-128875	-127931.01	-126673		15.65	-123056
GLW	-127937.825	14.36	15.29	15.36	-8362	-120078	-127854.98	-128440		15	-125430
JPM	261822.53	33.09	32.64	32.67	8019	265348.7	261740.16	261980.7		30.62	245541.8
LVS	164468.8128	47.57	47.6	48.1	3453	164259.2	164362.8	166089.3		45.27	156317.3
LYG	-127937.825	1.8	1.79	1.76	-71474	-128653	-127938.46	-125794		1.58	-112929
NOK	-4943.532708	6.5	6.51	6.56	-759	-4933.5	-4941.09	-4979.04		6.51	-4941.09
PFE	274057.889	19.64	19.81	19.79	13834	271699.8	274051.54	273774.9		19.53	270178
TWX	375385.2038	34.26	34.8	34.62	11143	381759.2	387776.4	385770.7		33.61	374516.2
WFC	145714.363	25.38	25.38	25.29	5741	145706.6	145706.58	145189.9		24.69	141745.3
Portfolio	**511751.3001**					**502881.9**	**524210.203**	**525888.2**			**533762.3**

Further tracking our stocks and portfolio returns along with the projected returns, we formed Table 6. The initial market values of our stocks this week use the prices when we adjusting each stock weight. This week's projected return is

$$\frac{534340.01}{524210.2} - 1 = 1.932\%.$$

We benefited from the $0.075 per share of dividend paid by GLW (short 8440 shares) on Nov 14th since the stock price must fall immediately after paying dividend. Actually, in the real market, if we short a stock we pay all the dividends issued during our holding period. So our actual return for this period is

$$\frac{533762.3 - 0.075 \times 8440}{524210.2} - 1 = 1.701\%.$$

Table 6: Projected Return VS Actual Return

Period 2	Projected Return($)	Initial Market Value($)	Projected Value($)	Market Value($)
BAC	-0.0229834	-127937.4	-124996.95	-121825
BP	0.00305614	257177.52	257963.49	249655
C	-0.0107073	-127944.8	-126574.88	-117813
EMC	-0.0031055	-66194.49	-65988.923	-62588.9
F	-1.02E-02	-127934.2	-126623.94	-116514
FCX	-0.0027261	-127928.3	-127579.58	-119095
GE	-0.005599	-127931	-127214.72	-123056
GLW	-0.0097509	-127855	-126608.28	-125430
JPM	-0.0090709	261740.16	259365.94	245541.8
LVS	0.00682224	164362.8	165484.12	156317.3
LYG	-0.021567	-127938.5	-125179.21	-112929
NOK	-0.0057118	-4941.09	-4912.8675	-4941.09
PFE	-0.0008779	274051.54	273810.96	270178
TWX	0.00092059	387776.4	388133.38	374516.2
WFC	-0.003055	145706.58	145261.45	141745.3
Portfolio		**524210.2**	**534340.01**	**533762.3**

3) Nov 21st (Monday) to Nov 25th (Friday)

This week we used the last Friday's market value $533,762.3 as our new total portfolio capital, and our program generated the following objective weights after adding the new week's data and looking back 6 months. Then we marked to market our new stock weights and thus rebalanced our portfolio as Table 7 shows.

Table 7: The Portfolio and Market Performance from Nov 21st, 2011 to Nov 25th, 2011

11/21/2011	Objective Weights($)	Average Purchasing Price($)	Price When Adjusting($)	Close Price($)	Position	Book Value($)	Weights When Adjusting($)	Market Value($)	11/25/2011	Close Price($)	Market Value($)
BAC	-133440.565	6.45	5.59	5.49	-23871	-153968	-133438.89	-131052		5.17	-123413
BP	213801.6615	43.24	41.63	41.7	5137	222123.9	213853.31	214212.9		39.41	202449.2
C	-133440.565	29.13	24.98	25	-5357	-156049	-133817.86	-133925		23.63	-126586
EMC	-92132.00015	23.93	22.64	22.74	-4069	-97371.2	-92122.16	-92529.1		21.88	-89029.7
F	-133440.565	10.99	9.93	10.05	-13438	-147684	-133439.34	-135052		9.75	-131021
FCX	-133440.565	39.6	35.81	36.14	-3729	-147668	-133535.49	-134766		33.82	-126115
GE	-133440.565	16.3	15.38	15.24	-8676	-141419	-133436.88	-132222		14.7	-127537
GLW	-133440.565	14.37	14.44	14.53	-9241	-132793	-133440.04	-134272		13.95	-128912
JPM	285475.276	32.56	29.8	29.91	9580	311924.8	285484	286537.8		28.48	272838.4
LVS	180864.2994	47.03	44.12	43.67	4095	192587.9	180671.4	178828.7		42.4	173628
LYG	-133440.565	1.73	1.46	1.48	-91398	-158119	-133441.08	-135269		1.4	-127957
NOK	-4017.304175	6.5	6.26	6.02	-642	-4173	-4018.92	-3864.84		5.29	-3396.18
PFE	326139.4028	19.57	19.26	18.96	16925	331222.3	325975.5	320898		18.45	312266.3
TWX	360813.41	34.26	32.77	33.18	11010	377202.6	360797.7	365311.8		32.36	356283.6
WFC	196901.4696	25.01	24.11	24.18	8167	204256.7	196906.37	197478.1		23.51	192006.2
Portfolio	**533762.26**					**500074**	**532997.62**	**530315.6**			**525505.1**

Further tracking our stocks and portfolio returns along with the projected returns, we formed Table 8. The initial market values of our stocks this week use the prices when we adjusting each stock weight. This week's projected return is

$$\frac{543672.61}{532997.62} - 1 = 2.003\%.$$

We received no dividend this week. Our actual return for this period is

$$\frac{525505.1}{532997.62} - 1 = -1.406\%.$$

Table 8: Projected Return VS Actual Return

Period 3	Projected Return($)	Initial Market Value($)	Projected Value($)	Market Value($)
BAC	-0.0242717	-133438.9	-130200.11	-123413
BP	0.00019783	213853.31	213895.62	202449.2
C	-0.0138837	-133817.9	-131959.97	-126586
EMC	-0.0055083	-92122.16	-91614.726	-89029.7
F	-0.013148	-133439.3	-131684.88	-131021
FCX	-0.007359	-133535.5	-132552.8	-126115
GE	-0.0061759	-133436.9	-132612.78	-127537
GLW	-0.0086203	-133440	-132289.75	-128912
JPM	-0.0109242	285484	282365.32	272838.4
LVS	0.00734444	180671.4	181998.33	173628
LYG	-0.0236982	-133441.1	-130278.77	-127957
NOK	-0.0051247	-4018.92	-3998.3244	-3396.18
PFE	-0.0006196	325975.5	325773.53	312266.3
TWX	-0.0007838	360797.7	360514.9	356283.6
WFC	-0.0029931	196906.37	196317.02	192006.2
Portfolio		**532997.62**	**543672.61**	**525505.1**

4) Nov 28th (Monday) to Dec 2nd (Friday)

This is our last holding week. This week we used the last Friday's market value $525,505.1 as our new total portfolio capital, and our program outputs the following objective weights after adding the new week's data and looking back 6 months. Then we marked to market our new stock weights and thus rebalanced our portfolio as Table 9 shows.

Table 9: The Portfolio and Market Performance from Nov 28th, 2011 to Dec 2nd, 2011

11/28/2011	Objective Weights($)	Average Purchasing Price($)	Price When Adjusting($)	Close Price($)	Position	Book Value($)	Weights When Adjusting($)	Market Value($)	12/2/2011	Close Price($)	Market Value($)
BAC	-131376.27	6.42	5.37	5.25	-24465	-157065	-131377.05	-128441		5.64	-137983
BP	215556.4944	43.18	40.87	40.82	5274	227731.3	215548.38	215284.7		43.29	228311.5
C	-131376.27	29.13	25.04	25.05	-5247	-152845	-131384.88	-131437		28.17	-147808
EMC	-131376.27	23.56	22.68	22.54	-5793	-136483	-131385.24	-130574		23.26	-134745
F	-131376.27	10.99	10.09	10	-13020	-143090	-131371.8	-130200		10.9	-141918
FCX	-131376.27	39.6	36.04	35.94	-3645	-144342	-131365.8	-131001		39.3	-143249
GE	-131376.27	16.3	15.16	14.8	-8664	-141223	-131346.24	-128227		16.09	-139404
GLW	-131376.27	14.37	14.94	14.78	-8794	-126370	-131382.36	-129975		13.22	-116257
JPM	244159.6417	32.56	29.62	29.16	8243	268392.1	244157.66	240365.9		32.33	266496.2
LVS	167017.0093	47.03	45.06	45.06	3707	174340.2	167037.42	167037.4		46.02	170596.1
LYG	-131376.27	1.73	1.45	1.44	-90604	-156745	-131375.8	-130470		1.52	-137718
NOK	-9778.067678	5.96	5.65	5.61	-1731	-10316.8	-9780.15	-9710.91		5.62	-9728.22
PFE	335024.5135	19.55	19.03	19.09	17605	344177.8	335023.15	336079.5		19.89	350163.5
TWX	388239.2995	34.2	33.23	33.31	11680	399456	388126.4	389060.8		34.41	401908.8
WFC	236296.3493	24.87	24.16	24.15	9776	243129.1	236188.16	236090.4		26.07	254860.3
Portfolio	**525505.08**					**488746.5**	**525311.85**	**533881.3**			**563527.4**

Further tracking our stocks and portfolio returns along with the projected returns, we formed Table 10. The initial market values of our stocks this week use the prices when we adjusting each stock weight. This week's projected return is

$$\frac{535941.22}{525311.85} - 1 = 2.023\%.$$

We received \$0.235 per share of dividend from TWX (long 11,010 shares) on Nov 28th and should pay the lender \$0.01 per share of dividend from BAC (short 24,465 shares) on Nov 30th. So our actual return for this period is

$$\frac{533881.3+0.235\times11010-0.01\times24465}{525311.85}-1=2.077\%.$$

Table 10: Projected Return VS Actual Return

Period 4	Projected Return($)	Initial Market Value($)	Projected Value($)	Market Value($)
BAC	-0.0293556	-131377.1	-127520.4	-128441
BP	-0.0044947	215548.38	214579.56	215284.7
C	-0.0197387	-131384.9	-128791.52	-131437
EMC	-0.0108109	-131385.2	-129964.85	-130574
F	-0.0165845	-131371.8	-129193.06	-130200
FCX	-0.0137391	-131365.8	-129560.95	-131001
GE	-0.0097182	-131346.2	-130069.8	-128227
GLW	-0.0126791	-131382.4	-129716.54	-129975
JPM	-0.0153791	244157.66	240402.74	240365.9
LVS	0.00058123	167037.42	167134.51	167037.4
LYG	-0.03188	-131375.8	-127187.54	-130470
NOK	-0.0056851	-9780.15	-9724.5493	-9710.91
PFE	-0.0043634	335023.15	333561.32	336079.5
TWX	-0.0032247	388126.4	386874.82	389060.8
WFC	-0.0045331	236188.16	235117.49	236090.4
Portfolio		**525311.85**	**535941.22**	**533881.3**

5. CONCLUSION

Comparing the projected returns and actual returns in Table 11 & Figure 4, we can see the actual returns are fluctuating around the expected returns calculated from our model. The average actual return is 2.04% which is very close to the average projected return of 2.00%. Our program seems to match the real market quite well.

Table 11

	Projected Return	Actual Return
1	2.03%	5.80%
2	1.93%	1.70%
3	2.00%	-1.41%
4	2.02%	2.08%
Average	2.00%	2.04%

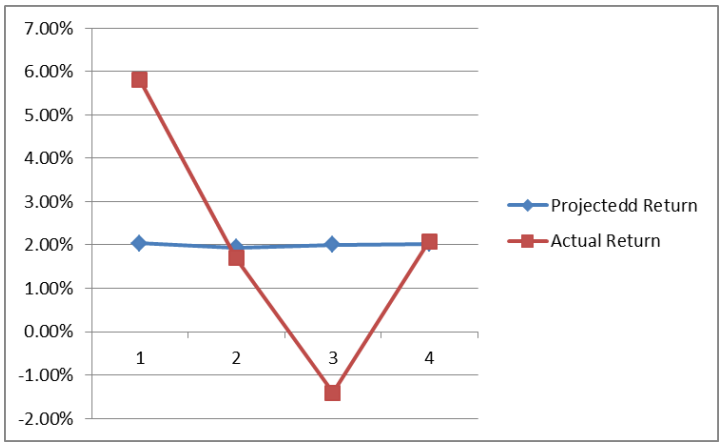


Figure 4

If we ignore the dividends to see the price returns of individual stocks, we notice that nearly all the stocks' prices drop as time goes on. But our portfolio nonetheless earns a positive return, which supports the rationale of our portfolio generation method.

Table 12: Weekly returns of each stock and the portfolio

	Week 1	Week 2	Week 3	Week 4
BAC	-0.0605144	-0.047776	-0.0751342	-0.02235
BP	0.01780759	-0.02925	-0.0533269	-0.00122
C	-0.0262284	-0.079187	-0.0540432	0.000399
EMC	0.00407498	-0.054469	-0.0335689	-0.00617
F	-0.0017921	-0.08927	-0.0181269	-0.00892
FCX	-0.016531	-0.069052	-0.0555711	-0.00277
GE	-0.0060976	-0.038107	-0.0442133	-0.02375
GLW	0.05779944	-0.018967	-0.0339335	-0.01071
JPM	-0.0098185	-0.061887	-0.0442953	-0.01553
LVS	-0.024816	-0.04895	-0.0389846	0
LYG	0.01666667	-0.117318	-0.0410959	-0.0069
NOK	0	0	-0.1549521	-0.00708
PFE	0.01782077	-0.014134	-0.0420561	0.003153
TWX	0.03013458	-0.034195	-0.0125114	0.002407
WFC	0.0106383	-0.027187	-0.0248859	-0.00041
Portfolio	**0.04850173**	**0.0182218**	**-0.0140574**	**0.016313**

Chapter 2: CAPM

1. INTRODUCTION AND OBJECTIVE:

Methods:

Here we use two models. The first is the security characteristic line model[9] and the second is a more elaborate model which allows a non-zero intercept.

Model I:

The security characteristic line[8] is defined by

$$R_{j,t} = \mu_{f,t} + \beta_j\left(R_{M,t} - \mu_{f,t}\right) + \varepsilon_{j,t}. \qquad (1)$$

Let

$$R^*_{j,t} = R_{j,t} - \mu_{f,t}$$

&

$$R^*_{M,t} = R_{M,t} - \mu_{f,t},$$

Then (1) can be written as

$$R^*_{j,t} = \beta_j R^*_{M,t} + \varepsilon_{j,t}.$$

For each stock's excess returns $R^*_{j,t}$s, we regress them on $R^*_{M,t}$ and force the intercept to be 0. Thus we can have the estimation of β_j and calculate the sample variance of residuals $\sigma^2_{\varepsilon,j}$ via $\varepsilon_{j,t}$s.

Model II:

If we allow the existence of intercepts, we can adjust our model I to be

$$R^*_{j,t} = \alpha_j + \beta_j R^*_{M,t} + \varepsilon_{j,t}.$$

For each stock's excess returns $R^*_{j,t}$s, we regress them on $R^*_{M,t}$. Thus we can have the estimation of α_j , β_j and calculate the sample variance of residuals $\sigma^2_{\varepsilon,j}$ via $\varepsilon_{j,t}$s.

2. RESULTS AND ANALYSIS:

Here we use the market data from 11th May, 2011 to 11th Nov, 2011. We chose S&P 500 index returns to be the market returns and the interest rate of Moody's AAA corporate bond to be the risk free

[9] Security characteristic line (SCL) is a regression line, plotting performance of a particular security or portfolio against that of the market portfolio at every point in time.

rate, which was then 4.09%. For each stock we regressed the excess stock returns on market returns, and the results are shown below.

Model I:

From the betas we obtained from those regressions, we can see that our stocks are all relatively risky or aggressive except for BP, GE, PFE and TWX, since most of the betas are larger than 1. Since our portfolio beta is less than 0.2, we can say that our portfolio is not that risky. Obviously our portfolio benefits from diversification. Also the smallest total risk, which is 0.03309, supports our view. Here the total risk is calculated as

$$\text{Total risk} = \beta_j^2 \sigma_M^2 + \sigma_\varepsilon^2,$$

σ_M^2 is the sample variance of the market returns and σ_ε^2 is the sample variance of the residuals from our regression.

When we consider the risk, we focus on the non-systematic risk, which can be reduced through diversification. Unfortunately from the adjusted R squared, we see that all of our stocks have significantly large non-systematic risk since little of their risks are explained by the CAPM regression model. Table 14 tells us that almost all of the absolute values of the residuals are even larger than predicted returns.

Table 13: Data from 'CAPM Project with no intercept.xlsx'

	Beta	Residual SS	Adjusted R Square	Sample Variance of Residual	Total Risk
BAC	1.6977241	0.949960944	0.103497035	0.007786565	0.093622
BP	0.9098477	0.254508989	0.110586405	0.002086139	0.048654
C	1.6340435	0.738661352	0.122088831	0.006054601	0.083433
EMC	1.1084557	0.259093801	0.156049608	0.00212372	0.050407
F	1.4465169	0.37055087	0.181437939	0.003037302	0.061218
FCX	1.6402922	0.653208713	0.137611951	0.00535417	0.079168
GE	0.8607029	0.214004708	0.117262087	0.001754137	0.044784
GLW	1.2216942	0.394620439	0.127305665	0.003234594	0.061166
JPM	1.2867844	0.302889863	0.176508514	0.002482704	0.05518
LVS	1.0876389	0.595960761	0.067811026	0.004884924	0.072709
LYG	1.8259667	1.010418594	0.1121011	0.00828212	0.097026
NOK	1.2214439	0.786804405	0.064659249	0.006449216	0.083401
PFE	0.8527403	0.112033881	0.203769536	0.000918311	0.034135
TWX	0.8515988	0.254060036	0.097585938	0.002082459	0.048256
WFC	1.2708248	0.266495803	0.192533247	0.002184392	0.052275
Portfolio	0.171046	0.132375881	0.000879285	0.001085048	0.03309

Table 14: Residual Output

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	0.003313	0.025715	0.783859
2	0.001456	0.019147	0.58364
3	-0.0063	0.021516	0.655852
4	0.001988	0.065373	1.992712
5	0.001058	0.063171	1.925585
6	-0.00109	0.065366	1.992518
7	0.003193	0.080164	2.443594
8	0.002735	0.054139	1.650292
9	-0.0048	0.090747	2.766191
10	-0.00425	0.063842	1.946064
11	4.74E-05	0.010188	0.310544
12	0.005846	-0.01087	-0.33139
13	0.001783	0.024448	0.745227
14	-0.00345	-0.02891	-0.88116
15	0.002183	-0.01914	-0.58343
16	0.003198	0.009449	0.28804
17	0.00076	0.023444	0.71463
18	-0.00218	0.03866	1.178456
… …	… …	… …	… …
109	-0.00241	-0.00949	-0.28926
110	0.001243	0.000136	0.004144
111	-0.00074	-0.01118	-0.3407
112	-0.00018	-0.00745	-0.22702
113	-0.00186	0.023814	0.725894
114	-0.00168	0.017503	0.533532
115	-0.00023	0.030842	0.940145
116	-0.00392	0.038124	1.162097
117	0.001793	0.018407	0.56109
118	0.000679	-0.02088	-0.63647
119	0.000657	-0.00992	-0.30239
120	0.000525	-0.00682	-0.20789
121	-0.00016	0.007671	0.233822
122	-0.00206	0.069097	2.106245
123	-0.00133	0.062802	1.914341

Model II:

Most of the results are similar to model I, but here we comment on the α's.

Seen from the first column of the table below, our α's are all significantly different from 0. That means under our assumptions of the model we use, those stocks are mispriced. We already know that if α

is positive, which means the stock is underpriced, we should long it and vice versa. Checking from our rebalanced weights that day, we can see three mismatches: JPM, PFE and WFC. This suggests that we should do more research on such mismatches and explain how they occur.

Table 15: Rebalanced Weights

BAC	-127938
BP	257165.2
C	-127938
EMC	-66354.4
F	-127938
FCX	-127938
GE	-127938
GLW	-127938
JPM	261822.5
LVS	164468.8
LYG	-127938
NOK	-4943.53
PFE	274057.9
TWX	375385.2
WFC	145714.4

Table 16: Data from 'CAPM Project with intercept.xlsx'

	Alpha	Beta	Residual SS	Adjusted R Square	Sample Variance of Residu	Total Risk
BAC	-0.02054	1.670908	0.898111234	0.106746187	0.007361567	0.0911559
BP	0.002803	0.913508	0.253543008	0.112693575	0.002078221	0.0485955
C	-0.00922	1.622007	0.728215659	0.122970088	0.005968981	0.0828381
EMC	-0.00328	1.104175	0.257772252	0.156896804	0.002112887	0.0502672
F	-0.00971	1.433843	0.358969912	0.185050076	0.002942376	0.0603352
FCX	-0.00278	1.636664	0.652259538	0.138304099	0.00534639	0.0790935
GE	-0.00611	0.852726	0.20941717	0.118505132	0.001716534	0.0443102
GLW	-0.01079	1.207608	0.380312921	0.129919672	0.003117319	0.0601033
JPM	-0.00804	1.276289	0.294947629	0.179431974	0.002417604	0.0545033
LVS	0.007377	1.097272	0.58927014	0.070403285	0.004830083	0.0723798
LYG	-0.02106	1.798469	0.95589984	0.115671016	0.007835245	0.0945163
NOK	-0.0065	1.212963	0.78161831	0.064657632	0.006406707	0.0831035
PFE	-0.00076	0.851746	0.111962523	0.205071184	0.000917726	0.0341177
TWX	0.000326	0.852024	0.25404699	0.098437788	0.002082352	0.0482577
WFC	-0.00215	1.268023	0.265929819	0.193661753	0.002179753	0.0522074
Portfolio	0.019424	0.196409	0.085994086	0.010127156	0.00070487	0.0267949

Table 17: Residual Output

Observation	*Predicted Y*	*Residuals*	*Standard Residuals*
1	0.023228	0.0058	0.218446
2	0.021096	-0.00049	-0.01858
3	0.012195	0.003025	0.113939
4	0.021707	0.045654	1.719572
5	0.020639	0.043589	1.641823
6	0.018169	0.046104	1.736542
7	0.023091	0.060266	2.269976
8	0.022565	0.034309	1.292285
9	0.013914	0.072035	2.71323
10	0.014544	0.045048	1.696774
11	0.019479	-0.00924	-0.34817
12	0.026137	-0.03116	-1.17377
13	0.021472	0.004759	0.179255
14	0.015465	-0.04782	-1.80119
15	0.021931	-0.03889	-1.46474
16	0.023097	-0.01045	-0.39357
17	0.020297	0.003907	0.147164
18	0.016918	0.01956	0.736727
… …	… …	… …	… …
109	0.016657	-0.02856	-1.07559
110	0.020851	-0.01947	-0.73345
111	0.01858	-0.03049	-1.14851
112	0.019214	-0.02684	-1.01113
113	0.017289	0.004665	0.175711
114	0.017491	-0.00167	-0.06296
115	0.019162	0.011452	0.431343
116	0.014927	0.01928	0.726194
117	0.021483	-0.00128	-0.04833
118	0.020204	-0.0404	-1.52188
119	0.020179	-0.02944	-1.10894
120	0.020028	-0.02632	-0.99144
121	0.01924	-0.01173	-0.44181
122	0.01706	0.049978	1.882463
123	0.017892	0.043575	1.641287

3. CONCLUSIONS AND IMPROVEMENTS:

Based on the results we have, we can conclude that stocks often have betas larger than one, and also they have larger nonsystematic risk than systematic risk since the residuals in our models are significantly large.

If we performed such analysis prior to initially forming our portfolio, we would like to seek stocks with smaller betas substitute for those with the highest ones. Furthermore, we may want to pick stocks with lower nonsystematic risks (or higher adjusted R square values).

Chapter 3: FACTOR MODEL

1. INTRODUCTION AND OBJECTIVE:

Consider the CAPM described in the prior chapter of our project,

$$R_{j,t} - \mu_{f,t} = \beta_j\left(R_{M,t} - \mu_{f,t}\right) + \varepsilon_{j,t}.$$

Here the variable $\left(R_{M,t} - \mu_{f,t}\right)$ which is the excess return on the market is sometimes called a factor or a risk factor. In the CAPM the market risk factor is the only source of risk besides the unique risk of each asset. Mispricing of an asset under only one unique factor may be uncomfortably serious. According to the idea of a factor model, we can generalize the CAPM by allowing more factors than simply the market risk and the unique risk of each asset. A multifactor model is

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}F_{1,t} + \cdots + \beta_{p,j}F_{p,t} + \varepsilon_{j,t}.$$ [10]

After determining the factors we use, we can form our models. The main objective of this project is to use a back-testing method to test our factor models. For example, we can test data from Jan 1st, 2009 to Dec 31st, 2009 in those models. Then we can use the factor coefficients generated to forecast the performance in the year 2010, during which we already know the real market data and can perform analysis and compare the effectiveness among different models.

2. STOCK AND FACTOR CHOICE:

The models we use in this project are based on two traditional models in portfolio analysis, the CAPM and the Fama & French model. In the CAPM, we use the market premium, $R_M - \mu_f$ as the main factor. In the Fama & French model, we use the market premium, small minus large (SML), and high minus low (HML) as the three main factors.[11] SML is the difference in returns on a portfolio of small stocks and a portfolio of large stocks. Here "small" and "large" refer to the size of the market equity which is the price of the stock times the number of shares outstanding. HML is the difference in returns on a portfolio of high book-to-market (BE/ME) stocks and a portfolio of low BE/ME stocks. Fama and French argue that most pricing anomalies that are inconsistent with the CAPM disappear in the tree-factor model. [12]

To further exploit the relationship between stock returns and the market conditions, we decided to add two other factors besides the market premium, SML and HML.

First, we considered the two methods of corporate financing, issuing stock or debt (or both). If a firm can issue corporate bonds at lower interest rates, it may have more strength to generate income and thus pay more dividends or do more investing. Such low interest rates might benefit the shareholders, make

[10] David Ruppert, Statistics and Finance: An Introduction p.242-243
[11] http://mba.tuck.dartmouth.edu/ Dartmouth College
[12] David Ruppert, Statistics and Finance: An Introduction

stock investment more attractive, and thus raise the stock returns. Thus we choose a type of corporate bond yield to be our first extended factor. Since the 15 stocks we chose were the top-15 stocks with largest market cap traded in the NYSE, we decided to use Moody's AAA Corporate Bond Yield.[13] In this paper, we denote this factor as the corporate premium (CP), which is the difference between the Corporate Bond Yield and the risk-free rate.

Second, we noticed that all of the 15 underlying firms whose stocks we chose have global businesses. Further, most of them already have foreign branches. We may want to see if the stock returns have an "international premium". To test our thinking, we chose the 3-Month Eurodollar Deposit Rate[14] as our second regression factor. In this paper we denote this factor as the international premium (IP), which is the difference between the 3-Month Eurodollar Deposit Rate and the risk-free rate.

Therefore, our project contains seven models, which are shown below:

The Fama & French Model:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}SML_t + \beta_{3,j}HML_t + \varepsilon_{j,t}$$

The Two-Factor Model with Corporate Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}CP_t + \varepsilon_{j,t}$$

The extended Fama & French Model with Corporate Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}SML_t + \beta_{3,j}HML_t + \beta_{4,j}CP_t + \varepsilon_{j,t}$$

The extended CAPM Model with International Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}IP_t + \varepsilon_{j,t}$$

The extended CAPM Model with both Corporate Premium factor and International Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}CP_t + \beta_{3,j}IP_t + \varepsilon_{j,t}$$

The extended Fama & French Model with International Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}SML_t + \beta_{3,j}HML_t + \beta_{4,j}IP_t + \varepsilon_{j,t}$$

The extended Fama & French Model with both Corporate Premium factor and International Premium factor:

$$R_{j,t} - \mu_{f,t} = \beta_{0,j} + \beta_{1,j}(R_{M,t} - \mu_{f,t}) + \beta_{2,j}SML_t + \beta_{3,j}HML_t + \beta_{4,j}CP_t + \beta_{5,j}IP_t + \varepsilon_{j,t}$$

Here $R_{j,t}$ represents the jth stock's return at time t, $\mu_{f,t}$ represents the risk-free rate at time t, and $R_{M,t}$ represents the market rate of return at time t.

[13] http://research.stlouisfed.org/fred2/ Economic Research
[14] http://research.stlouisfed.org/fred2/ Economic Research

To test our models, we choose 15 stocks exchanged in the NYSE. Those stocks are chosen from different sectors for diversification. They also have the largest market caps to insure marketability and stability. Some of the firms are headquartered outside the U.S. (e.g. BP, NOK), which makes our international premium factor more meaningful.

Table 18: Symbols, Sectors and Market Capitalizations for Stocks

Name	Symbol	Sector	Market Cap
Bank of America Corporation	BAC	Financial	51.17B
Citigroup, Inc.	C	Financial	82.36B
JP Morgan Chase & Co.	JPM	Financial	122.78B
Lloyds Banking Group Plc Americ	LYG	Financial	26.67B
Wells Fargo & Company	WFC	Financial	137.48B
EMC Corporation	EMC	Technology	47.45B
Corning Incorporated	GLW	Technology	20.78B
Nokia Corporation Sponsored Ame	NOK	Technology	20.85B
BP p.l.c.	BP	Basic Materials	136.78B
Freeport-McMoRan Copper & Gold	FCX	Basic Materials	37.25B
Las Vegas Sands Corp.	LVS	Services	33.58B
Time Warner Inc. New	TWX	Services	34.44B
Ford Motor Company	F	Consumer Goods	41.42B
General Electric Company	GE	Industrial Goods	169.87B
Pfizer, Inc.	PFE	Healthcare	152.89B

3. MATLAB OUTPUTS:

Here we select our data from the trading days from Mar 1st, 2010 to Dec 31st, 2010. During the time range we calculated the daily returns of each of the 15 stocks and found the values of each factor.

Our regression program runs each model individually through 15 assets. For each model, we computed the regression coefficients, $\beta_{i,j}$'s, which represents the loading of the j^{th} asset on the i^{th} factor. Also the function 'regress' in the Statistics Toolbox of MATLAB returns the R-square statistic, F statistic and p value for the full model, and an estimate of the error variance, which helps us to check if our models perform well.

In order to form our optimal tangency portfolios, we computed the asset return covariance matrix (the diagonal elements are the return variance) specific to each model, and gave out the expected daily returns. Once we generated the data, we used our MATLAB program to form the Markowitz tangency portfolios. We also showed the optimal weights for those stocks.

The data of each model are shown below in the same order as the seven models shown above.

Table 19: The Fama & French Model (FFM)

	β0	β1	β2	β3	R-squared	F-value	p-value	error variance
BAC	-0.00102	0.010914	-0.00588	0.017928	0.701562	164.5549	7.05E-55	0.000158
BP	-0.00064	0.009757	-0.009	0.006178	0.185575	15.95024	2.22E-09	0.000674
C	0.001521	0.008794	-0.00115	0.017541	0.538545	81.69396	4.63E-35	0.000269
EMC	0.000285	0.011072	0.001198	-0.00449	0.560012	89.09539	3.17E-37	0.000116
F	0.000864	0.011803	0.003996	0.005431	0.547961	84.85381	5.36E-36	0.000253
FCX	0.001351	0.016703	-0.00237	0.003898	0.618964	113.7095	9.19E-44	0.000266
GE	5.45E-05	0.011398	-0.00158	0.004406	0.725143	184.6776	1.26E-58	8.39E-05
GLW	-0.0005	0.013741	-0.00129	-0.00279	0.545234	83.92547	1.01E-35	0.000191
JPM	0.000136	0.009036	-0.00626	0.016298	0.735759	194.9098	2.04E-60	9.63E-05
LVS	0.003924	0.017076	0.001384	0.005922	0.465065	60.85697	2.36E-28	0.000614
LYG	0.000342	0.018708	-0.00715	0.002631	0.514193	74.09015	1.00E-32	0.000439
NOK	-0.00178	0.015253	-0.00901	-0.00802	0.354882	38.50733	7.17E-20	0.000344
PFE	-0.00042	0.008931	-0.00374	-0.00291	0.435564	54.01752	6.40E-26	0.0001
TWX	-0.00018	0.009901	0.000694	0.000655	0.662899	137.6529	2.46E-49	7.71E-05
WFC	0.000372	0.011944	-0.00548	0.013047	0.704101	166.5674	2.88E-55	0.00014

Table 20: Covariance Matrix

0.000521	0.000227	0.000334	0.000197	0.000309	0.000376	0.000272	0.000259	0.00031	0.000415	0.00039	0.000218	0.000149	0.000216	0.000346
0.000227	0.000816	0.000202	0.000126	0.000187	0.000238	0.000171	0.000167	0.000194	0.000256	0.000254	0.000154	0.000101	0.000135	0.000217
0.000334	0.000202	0.000576	0.000184	0.000292	0.000347	0.000251	0.000239	0.000282	0.000387	0.000355	0.000192	0.000134	0.000201	0.000316
0.000197	0.000126	0.000184	0.00026	0.000203	0.000242	0.000172	0.00018	0.000163	0.000269	0.000247	0.000152	0.000102	0.000146	0.000195
0.000309	0.000187	0.000292	0.000203	0.000552	0.000353	0.000253	0.000254	0.000257	0.000397	0.000357	0.000203	0.00014	0.000211	0.000299
0.000376	0.000238	0.000347	0.000242	0.000353	0.000689	0.000305	0.000307	0.000315	0.000472	0.000439	0.000261	0.000175	0.000251	0.000365
0.000272	0.000171	0.000251	0.000172	0.000253	0.000305	0.000301	0.000218	0.000228	0.000338	0.000314	0.000184	0.000124	0.000179	0.000263
0.000259	0.000167	0.000239	0.00018	0.000254	0.000307	0.000218	0.000414	0.000216	0.00034	0.000317	0.000194	0.000129	0.000183	0.000255
0.00031	0.000194	0.000282	0.000163	0.000257	0.000315	0.000228	0.000216	0.000359	0.000346	0.000329	0.000183	0.000125	0.00018	0.000293
0.000415	0.000256	0.000387	0.000269	0.000397	0.000472	0.000338	0.00034	0.000346	0.001132	0.000481	0.000279	0.00019	0.00028	0.000402
0.00039	0.000254	0.000355	0.000247	0.000357	0.000439	0.000314	0.000317	0.000329	0.000481	0.000892	0.000279	0.000184	0.000257	0.000379
0.000218	0.000154	0.000192	0.000152	0.000203	0.000261	0.000184	0.000194	0.000183	0.000279	0.000279	0.000526	0.000118	0.000152	0.000217
0.000149	0.000101	0.000134	0.000102	0.00014	0.000175	0.000124	0.000129	0.000125	0.00019	0.000184	0.000118	0.000175	0.000103	0.000147
0.000216	0.000135	0.000201	0.000146	0.000211	0.000251	0.000179	0.000183	0.00018	0.00028	0.000257	0.000152	0.000103	0.000226	0.000211
0.000346	0.000217	0.000316	0.000195	0.000299	0.000365	0.000263	0.000255	0.000293	0.000402	0.000379	0.000217	0.000147	0.000211	0.000468

Table 21: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.9219	-0.0007
BP	-0.6534	-0.0003
C	2.5569	0.0019
EMC	3.4346	0.0015
F	1.9042	0.002
FCX	3.1989	0.0027
GE	0.0249	0.0009
GLW	-1.1474	0.0008
JPM	-2.1006	0.0003
LVS	4.1626	0.0054
LYG	0.4421	0.0016
NOK	-2.6665	-0.0006
PFE	-0.6258	0.0003
TWX	-0.6394	0.0008
WFC	0.0307	0.0009

Table 22: The Two-Factor Model with Corporate Premium factor (CAPMC)

	β0	β1	β2	R-squared	F-value	p-value	error variance
BAC	-0.02508	0.014717	0.004732	0.598646	157.3603	1.49E-42	0.000211
BP	0.026816	0.009443	-0.00575	0.154948	19.34437	1.93E-08	0.000696
C	-0.02047	0.013672	0.004337	0.467248	92.52839	1.41E-29	0.00031
EMC	-0.00733	0.010004	0.001613	0.548319	128.0718	3.84E-37	0.000119
F	0.008075	0.01445	-0.00151	0.534727	121.2488	8.77E-36	0.000259
FCX	0.05585	0.01746	-0.01124	0.62791	178.0334	5.05E-46	0.000259
GE	-0.01247	0.012276	0.002519	0.715277	265.0358	2.76E-58	8.65E-05
GLW	-0.00372	0.012573	0.00068	0.541988	124.8434	1.67E-36	0.000191
JPM	0.002481	0.012346	-0.00068	0.599774	158.1009	1.10E-42	0.000145
LVS	0.00838	0.019207	-0.00097	0.460712	90.12846	5.09E-29	0.000616
LYG	-0.01179	0.017663	0.002425	0.498322	104.7942	2.48E-32	0.000452
NOK	0.042732	0.010751	-0.00911	0.314947	48.50269	4.67E-18	0.000364
PFE	0.017908	0.007189	-0.00376	0.416655	75.3535	2.02E-25	0.000103
TWX	-0.01693	0.010214	0.003438	0.666264	210.6178	5.24E-51	7.60E-05
WFC	-0.02261	0.014394	0.004561	0.637536	185.5634	3.18E-47	0.000171

Table 23: Covariance Matrix

0.000521	0.000194	0.00029	0.000211	0.000302	0.000358	0.000259	0.000264	0.000258	0.000402	0.000372	0.000219	0.000148	0.000217	0.000305
0.000194	0.000816	0.00018	0.000132	0.000192	0.000234	0.000162	0.000166	0.000164	0.000255	0.000233	0.000144	9.61E-05	0.000134	0.00019
0.00029	0.00018	0.000576	0.000196	0.00028	0.000332	0.000241	0.000245	0.00024	0.000373	0.000345	0.000203	0.000137	0.000201	0.000283
0.000211	0.000132	0.000196	0.00026	0.000205	0.000244	0.000175	0.000179	0.000175	0.000273	0.000252	0.000149	0.000101	0.000146	0.000206
0.000302	0.000192	0.00028	0.000205	0.000552	0.000354	0.000251	0.000257	0.000252	0.000392	0.000361	0.000218	0.000146	0.000209	0.000295
0.000358	0.000234	0.000332	0.000244	0.000354	0.000689	0.000299	0.000307	0.000302	0.00047	0.000431	0.000267	0.000178	0.000248	0.00035
0.000259	0.000162	0.000241	0.000175	0.000251	0.000299	0.000301	0.00022	0.000215	0.000335	0.000309	0.000183	0.000124	0.00018	0.000253
0.000264	0.000166	0.000245	0.000179	0.000257	0.000307	0.00022	0.000414	0.00022	0.000342	0.000316	0.000188	0.000127	0.000183	0.000258
0.000258	0.000164	0.00024	0.000175	0.000252	0.000302	0.000215	0.00022	0.000359	0.000335	0.000309	0.000186	0.000125	0.000179	0.000253
0.000402	0.000255	0.000373	0.000273	0.000392	0.00047	0.000335	0.000342	0.000335	0.001132	0.000481	0.000289	0.000194	0.000279	0.000393
0.000372	0.000233	0.000345	0.000252	0.000361	0.000431	0.000309	0.000316	0.000309	0.000481	0.000892	0.000264	0.000178	0.000258	0.000364
0.000219	0.000144	0.000203	0.000149	0.000218	0.000267	0.000183	0.000188	0.000186	0.000289	0.000264	0.000526	0.00011	0.000152	0.000214
0.000148	9.61E-05	0.000137	0.000101	0.000146	0.000178	0.000124	0.000127	0.000125	0.000194	0.000178	0.00011	0.000175	0.000103	0.000145
0.000217	0.000134	0.000201	0.000146	0.000209	0.000248	0.00018	0.000183	0.000179	0.000279	0.000258	0.000152	0.000103	0.000226	0.000212
0.000305	0.00019	0.000283	0.000206	0.000295	0.00035	0.000253	0.000258	0.000253	0.000393	0.000364	0.000214	0.000145	0.000212	0.000468

Table 24: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.1397	-0.0007
BP	-0.9557	-0.0003
C	1.8539	0.0019
EMC	4.417	0.0015
F	2.3007	0.002
FCX	3.1501	0.0027
GE	-0.0488	0.0009
GLW	-0.7952	0.0008
JPM	-2.9861	0.0003
LVS	4.2157	0.0054
LYG	0.1996	0.0016
NOK	-2.771	-0.0006
PFE	-0.8533	0.0003
TWX	0.3581	0.0008
WFC	-0.9453	0.0009

Table 25: The extended Fama & French Model with Corporate Premium factor (EFFMC)

	β0	β1	β2	β3	β4	R-squared	F-value	p-value	error variance
BAC	-0.00761	0.01096	-0.00594	0.017744	0.001353	0.701829	122.9847	8.97E-54	0.000158
BP	0.030524	0.009537	-0.00874	0.007049	-0.00639	0.189368	12.20585	6.17E-09	0.000674
C	-0.00091	0.008811	-0.00118	0.017473	0.000498	0.538577	60.9867	4.53E-34	0.000271
EMC	-0.01236	0.011161	0.001091	-0.00484	0.002594	0.56197	67.03404	2.06E-36	0.000116
F	0.016951	0.011689	0.004132	0.00588	-0.0033	0.549455	63.72071	3.82E-35	0.000253
FCX	0.061337	0.016279	-0.00186	0.005573	-0.0123	0.635599	91.13587	1.03E-44	0.000256
GE	-0.0085	0.011458	-0.00166	0.004168	0.001754	0.725916	138.3853	1.39E-57	8.41E-05
GLW	-0.00786	0.013793	-0.00135	-0.003	0.001509	0.545651	62.74972	9.14E-35	0.000192
JPM	0.018794	0.008905	-0.00611	0.016819	-0.00383	0.738844	147.8219	9.11E-60	9.57E-05
LVS	0.016367	0.016988	0.001489	0.006269	-0.00255	0.4655	45.50496	1.85E-27	0.000617
LYG	-0.01285	0.018801	-0.00726	0.002262	0.002706	0.514815	55.44086	8.24E-32	0.000441
NOK	0.030075	0.015029	-0.00874	-0.00713	-0.00653	0.361026	29.52173	1.82E-19	0.000343
PFE	0.013158	0.008835	-0.00362	-0.00253	-0.00278	0.438914	40.87294	2.78E-25	0.0001
TWX	-0.01642	0.010016	0.000557	0.000202	0.003331	0.666624	104.4798	9.90E-49	7.67E-05
WFC	-0.01074	0.012023	-0.00557	0.012737	0.002279	0.704942	124.8338	3.01E-54	0.000141

Table 26: Covariance Matrix

0.000521	0.000227	0.000334	0.000197	0.000309	0.000375	0.000272	0.000259	0.00031	0.000414	0.000391	0.000217	0.000148	0.000216	0.000346
0.000227	0.000816	0.000201	0.000124	0.000188	0.000244	0.00017	0.000166	0.000196	0.000258	0.000253	0.000157	0.000102	0.000133	0.000216
0.000334	0.000201	0.000576	0.000184	0.000291	0.000346	0.000251	0.000239	0.000282	0.000387	0.000355	0.000192	0.000134	0.000201	0.000316
0.000197	0.000124	0.000184	0.00026	0.000202	0.00024	0.000172	0.00018	0.000162	0.000269	0.000248	0.000151	0.000101	0.000146	0.000196
0.000309	0.000188	0.000291	0.000202	0.000552	0.000356	0.000253	0.000254	0.000258	0.000398	0.000356	0.000204	0.000141	0.00021	0.000299
0.000375	0.000244	0.000346	0.00024	0.000356	0.000689	0.000303	0.000306	0.000319	0.000474	0.000437	0.000267	0.000178	0.000248	0.000363
0.000272	0.00017	0.000251	0.000172	0.000253	0.000303	0.000301	0.000219	0.000228	0.000337	0.000314	0.000183	0.000124	0.00018	0.000263
0.000259	0.000166	0.000239	0.00018	0.000254	0.000306	0.000219	0.000414	0.000215	0.000339	0.000317	0.000194	0.000129	0.000183	0.000255
0.00031	0.000196	0.000282	0.000162	0.000258	0.000319	0.000228	0.000215	0.000359	0.000347	0.000328	0.000185	0.000126	0.000179	0.000292
0.000414	0.000258	0.000387	0.000269	0.000398	0.000474	0.000337	0.000339	0.000347	0.001132	0.000481	0.000281	0.000191	0.000279	0.000402
0.000391	0.000253	0.000355	0.000248	0.000356	0.000437	0.000314	0.000317	0.000328	0.000481	0.000892	0.000278	0.000184	0.000257	0.00038
0.000217	0.000157	0.000192	0.000151	0.000204	0.000267	0.000183	0.000194	0.000185	0.000281	0.000278	0.000526	0.000119	0.00015	0.000216
0.000148	0.000102	0.000134	0.000101	0.000141	0.000178	0.000124	0.000129	0.000126	0.000191	0.000184	0.000119	0.000175	0.000102	0.000146
0.000216	0.000133	0.000201	0.000146	0.00021	0.000248	0.00018	0.000183	0.000179	0.000279	0.000257	0.00015	0.000102	0.000226	0.000211
0.000346	0.000216	0.000316	0.000196	0.000299	0.000363	0.000263	0.000255	0.000292	0.000402	0.00038	0.000216	0.000146	0.000211	0.000468

Table 27: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.885	-0.0007
BP	-0.6686	-0.0003
C	2.5495	0.0019
EMC	3.473	0.0015
F	1.8661	0.002
FCX	3.2271	0.0027
GE	0.0574	0.0009
GLW	-1.1363	0.0008
JPM	-2.182	0.0003
LVS	4.1352	0.0054
LYG	0.4484	0.0016
NOK	-2.7166	-0.0006
PFE	-0.6636	0.0003
TWX	-0.5662	0.0008
WFC	0.0615	0.0009

Table 28: The extended CAPM Model with International Premium factor (CAPMI)

	β0	β1	β2	R-squared	F-value	p-value	error variance
BAC	-0.00023	0.014781	-0.00389	0.595489	155.3087	3.40E-42	0.000213
BP	0.000291	0.009337	-0.00317	0.151862	18.8901	2.84E-08	0.000698
C	0.001021	0.01374	-0.00079	0.464691	91.58261	2.33E-29	0.000311
EMC	-0.00048	0.010039	0.002182	0.547671	127.7375	4.47E-37	0.000119
F	-0.00085	0.014438	0.003388	0.534559	121.1667	9.11E-36	0.000259
FCX	-0.0004	0.017287	0.003243	0.613628	167.5527	2.69E-44	0.000269
GE	0.002937	0.012292	-0.00678	0.714772	264.3798	3.33E-58	8.67E-05
GLW	-0.00137	0.012592	0.002088	0.54198	124.8392	1.67E-36	0.000191
JPM	-0.0033	0.012355	0.005395	0.600284	158.4377	9.65E-43	0.000145
LVS	-0.0009	0.019227	0.009898	0.4613	90.34178	4.54E-29	0.000616
LYG	-0.01992	0.017862	0.043108	0.51351	111.3597	9.69E-34	0.000438
NOK	-0.00373	0.010618	0.004521	0.302854	45.83136	2.96E-17	0.00037
PFE	-0.00434	0.007159	0.00856	0.413489	74.37741	3.57E-25	0.000104
TWX	-0.003	0.010293	0.006072	0.663382	207.9115	1.30E-50	7.67E-05
WFC	0.006637	0.014413	-0.01521	0.637771	185.752	2.97E-47	0.000171

Table 29: Covariance Matrix

0.000521	0.000196	0.000288	0.00021	0.000302	0.000362	0.000258	0.000264	0.000258	0.000402	0.00037	0.000222	0.000149	0.000215	0.000304
0.000196	0.000816	0.000182	0.000133	0.000191	0.000229	0.000163	0.000167	0.000163	0.000254	0.000233	0.00014	9.42E-05	0.000136	0.000192
0.000288	0.000182	0.000576	0.000195	0.000281	0.000336	0.00024	0.000245	0.00024	0.000374	0.000344	0.000206	0.000139	0.0002	0.000282
0.00021	0.000133	0.000195	0.00026	0.000205	0.000245	0.000175	0.000179	0.000175	0.000273	0.000252	0.000151	0.000101	0.000146	0.000205
0.000302	0.000191	0.000281	0.000205	0.000552	0.000353	0.000251	0.000257	0.000252	0.000392	0.000363	0.000217	0.000146	0.00021	0.000295
0.000362	0.000229	0.000336	0.000245	0.000353	0.000689	0.000301	0.000308	0.000302	0.00047	0.000434	0.000259	0.000175	0.000251	0.000354
0.000258	0.000163	0.00024	0.000175	0.000251	0.000301	0.000301	0.000219	0.000215	0.000334	0.000307	0.000185	0.000124	0.000179	0.000253
0.000264	0.000167	0.000245	0.000179	0.000257	0.000308	0.000219	0.000414	0.00022	0.000342	0.000316	0.000189	0.000127	0.000183	0.000258
0.000258	0.000163	0.00024	0.000175	0.000252	0.000302	0.000215	0.00022	0.000359	0.000336	0.000311	0.000185	0.000125	0.00018	0.000252
0.000402	0.000254	0.000374	0.000273	0.000392	0.00047	0.000334	0.000342	0.000336	0.001132	0.000484	0.000288	0.000194	0.00028	0.000392
0.00037	0.000233	0.000344	0.000252	0.000363	0.000434	0.000307	0.000316	0.000311	0.000484	0.000892	0.000267	0.000181	0.00026	0.000358
0.000222	0.00014	0.000206	0.000151	0.000217	0.000259	0.000185	0.000189	0.000185	0.000288	0.000267	0.000526	0.000107	0.000154	0.000217
0.000149	9.42E-05	0.000139	0.000101	0.000146	0.000175	0.000124	0.000127	0.000125	0.000194	0.000181	0.000107	0.000175	0.000104	0.000145
0.000215	0.000136	0.0002	0.000146	0.00021	0.000251	0.000179	0.000183	0.00018	0.00028	0.00026	0.000154	0.000104	0.000226	0.00021
0.000304	0.000192	0.000282	0.000205	0.000295	0.000354	0.000253	0.000258	0.000252	0.000392	0.000358	0.000217	0.000145	0.00021	0.000468

Table 30: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.1427	-0.0007
BP	-0.9378	-0.0003
C	1.8414	0.0019
EMC	4.4123	0.0015
F	2.3309	0.002
FCX	3.1438	0.0027
GE	-0.0277	0.0009
GLW	-0.7995	0.0008
JPM	-3.0041	0.0003
LVS	4.2487	0.0054
LYG	0.1483	0.0016
NOK	-2.6949	-0.0006
PFE	-0.8341	0.0003
TWX	0.243	0.0008
WFC	-0.9276	0.0009

Table 31: The extended CAPM Model with both Corporate Premium factor and International Premium factor (CAPMCI)

	β0	β1	β2	β3	R-squared	F-value	p-value	error variance
BAC	-0.02562	0.014718	0.004787	0.000604	0.598651	104.4118	2.11E-41	0.000212
BP	0.035243	0.009423	-0.00659	-0.00935	0.155688	12.90771	9.00E-08	0.000698
C	-0.02369	0.01368	0.004659	0.003576	0.467401	61.43105	1.49E-28	0.000311
EMC	-0.01096	0.010013	0.001977	0.004036	0.548751	85.12483	4.46E-36	0.000119
F	0.006136	0.014455	-0.00132	0.002152	0.534785	80.46816	1.08E-34	0.00026
FCX	0.063032	0.017442	-0.01196	-0.00797	0.628546	118.4487	6.38E-45	0.00026
GE	-0.00812	0.012265	0.002085	-0.00482	0.71581	176.3141	4.19E-57	8.68E-05
GLW	-0.0064	0.012579	0.000948	0.002977	0.542136	82.88384	2.05E-35	0.000192
JPM	-0.00221	0.012358	-0.00021	0.005201	0.600293	105.1283	1.37E-41	0.000146
LVS	-0.00047	0.019228	-8.09E-05	0.009822	0.4613	59.94252	4.90E-28	0.000619
LYG	-0.05645	0.017772	0.006887	0.049567	0.517334	75.0279	5.08E-33	0.000437
NOK	0.046691	0.010742	-0.00951	-0.00439	0.3152	32.21966	3.57E-17	0.000365
PFE	0.01295	0.007201	-0.00326	0.005503	0.417847	50.24323	1.61E-24	0.000104
TWX	-0.02608	0.010236	0.004352	0.010153	0.669416	141.7467	3.18E-50	7.57E-05
WFC	-0.01186	0.014368	0.003486	-0.01194	0.639639	124.2495	2.67E-46	0.000171

Table 32: Covariance Matrix

0.000521	0.000194	0.00029	0.000211	0.000302	0.000358	0.000259	0.000264	0.000258	0.000402	0.000372	0.000219	0.000148	0.000217	0.000305
0.000194	0.000816	0.00018	0.000132	0.000192	0.000234	0.000162	0.000166	0.000163	0.000254	0.00023	0.000145	9.57E-05	0.000134	0.00019
0.00029	0.00018	0.000576	0.000196	0.00028	0.000332	0.000241	0.000245	0.00024	0.000373	0.000347	0.000203	0.000138	0.000201	0.000283
0.000211	0.000132	0.000196	0.00026	0.000205	0.000244	0.000175	0.000179	0.000175	0.000273	0.000253	0.000149	0.000101	0.000147	0.000206
0.000302	0.000192	0.00028	0.000205	0.000552	0.000354	0.000251	0.000257	0.000252	0.000392	0.000362	0.000218	0.000146	0.00021	0.000295
0.000358	0.000234	0.000332	0.000244	0.000354	0.000689	0.000299	0.000307	0.000302	0.00047	0.000428	0.000268	0.000177	0.000248	0.000351
0.000259	0.000162	0.000241	0.000175	0.000251	0.000299	0.000301	0.00022	0.000215	0.000334	0.000308	0.000183	0.000123	0.00018	0.000254
0.000264	0.000166	0.000245	0.000179	0.000257	0.000307	0.00022	0.000414	0.00022	0.000342	0.000317	0.000188	0.000127	0.000183	0.000258
0.000258	0.000163	0.00024	0.000175	0.000252	0.000302	0.000215	0.00022	0.000359	0.000336	0.000311	0.000186	0.000125	0.00018	0.000252
0.000402	0.000254	0.000373	0.000273	0.000392	0.00047	0.000334	0.000342	0.000336	0.001132	0.000484	0.000289	0.000194	0.00028	0.000392
0.000372	0.00023	0.000347	0.000253	0.000362	0.000428	0.000308	0.000317	0.000311	0.000484	0.000892	0.000263	0.00018	0.000262	0.00036
0.000219	0.000145	0.000203	0.000149	0.000218	0.000268	0.000183	0.000188	0.000186	0.000289	0.000263	0.000526	0.00011	0.000151	0.000214
0.000148	9.57E-05	0.000138	0.000101	0.000146	0.000177	0.000123	0.000127	0.000125	0.000194	0.00018	0.00011	0.000175	0.000103	0.000144
0.000217	0.000134	0.000201	0.000147	0.00021	0.000248	0.00018	0.000183	0.00018	0.00028	0.000262	0.000151	0.000103	0.000226	0.000211
0.000305	0.00019	0.000283	0.000206	0.000295	0.000351	0.000254	0.000258	0.000252	0.000392	0.00036	0.000214	0.000144	0.000211	0.000468

Table 33: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.1368	-0.0007
BP	-0.9491	-0.0003
C	1.8529	0.0019
EMC	4.4175	0.0015
F	2.3009	0.002
FCX	3.1708	0.0027
GE	-0.0117	0.0009
GLW	-0.7968	0.0008
JPM	-2.9985	0.0003
LVS	4.2145	0.0054
LYG	0.1596	0.0016
NOK	-2.7642	-0.0006
PFE	-0.8649	0.0003
TWX	0.3203	0.0008
WFC	-0.9145	0.0009

Table 34: The extended Fama & French Model with International Premium factor (EFFMI)

	β0	β1	β2	β3	β4	R-squared	F-value	p-value	error variance
BAC	0.000968	0.010939	-0.00598	0.017867	-0.00429	0.701824	122.982	8.99E-54	0.000158
BP	0.003232	0.009807	-0.00918	0.00606	-0.00837	0.186211	11.95583	9.11E-09	0.000676
C	0.000512	0.008781	-0.00111	0.017572	0.002182	0.538606	60.99373	4.50E-34	0.000271
EMC	-0.00069	0.01106	0.001243	-0.00446	0.002107	0.560139	66.53741	3.17E-36	0.000117
F	-0.00261	0.011758	0.004158	0.005537	0.007514	0.548718	63.53134	4.53E-35	0.000254
FCX	0.000219	0.016688	-0.00231	0.003933	0.002448	0.619028	84.89919	1.05E-42	0.000267
GE	0.003331	0.01144	-0.00174	0.004306	-0.00708	0.726376	138.7055	1.17E-57	8.40E-05
GLW	-0.00077	0.013737	-0.00128	-0.00278	0.000587	0.545241	62.64592	1.00E-34	0.000192
JPM	-0.00196	0.009009	-0.00617	0.016362	0.004533	0.736182	145.8034	2.62E-59	9.66E-05
LVS	-0.0018	0.017002	0.00165	0.006097	0.01237	0.466065	45.60827	1.66E-27	0.000616
LYG	-0.01785	0.018474	-0.0063	0.003189	0.039331	0.527029	58.22187	5.87E-33	0.00043
NOK	-0.0001	0.015275	-0.00909	-0.00808	-0.00364	0.355068	28.76633	4.73E-19	0.000346
PFE	-0.00292	0.008899	-0.00362	-0.00283	0.005394	0.436791	40.52202	4.11E-25	0.000101
TWX	-0.00333	0.009861	0.000841	0.000752	0.006827	0.664427	103.454	1.96E-48	7.72E-05
WFC	0.008046	0.012043	-0.00584	0.012811	-0.01659	0.708458	126.9693	8.64E-55	0.000139

Table 35: Covariance Matrix

0.000521	0.000228	0.000334	0.000197	0.000309	0.000376	0.000272	0.000259	0.00031	0.000414	0.000389	0.000218	0.000148	0.000215	0.000346
0.000228	0.000816	0.000202	0.000125	0.000186	0.000238	0.000172	0.000167	0.000194	0.000256	0.000252	0.000155	0.0001	0.000135	0.000218
0.000334	0.000202	0.000576	0.000184	0.000292	0.000347	0.000251	0.000239	0.000283	0.000387	0.000355	0.000192	0.000134	0.000201	0.000316
0.000197	0.000125	0.000184	0.00026	0.000203	0.000242	0.000172	0.00018	0.000163	0.00027	0.000248	0.000152	0.000102	0.000146	0.000195
0.000309	0.000186	0.000292	0.000203	0.000552	0.000353	0.000253	0.000254	0.000257	0.000398	0.000359	0.000202	0.00014	0.000211	0.000298
0.000376	0.000238	0.000347	0.000242	0.000353	0.000689	0.000305	0.000307	0.000315	0.000472	0.00044	0.000261	0.000175	0.000251	0.000365
0.000272	0.000172	0.000251	0.000172	0.000253	0.000305	0.000301	0.000218	0.000228	0.000337	0.000312	0.000184	0.000124	0.000179	0.000264
0.000259	0.000167	0.000239	0.00018	0.000254	0.000307	0.000218	0.000414	0.000216	0.00034	0.000317	0.000194	0.000129	0.000183	0.000255
0.00031	0.000194	0.000283	0.000163	0.000257	0.000315	0.000228	0.000216	0.000359	0.000347	0.00033	0.000183	0.000125	0.00018	0.000292
0.000414	0.000256	0.000387	0.00027	0.000398	0.000472	0.000337	0.00034	0.000347	0.001132	0.000485	0.000279	0.000191	0.00028	0.0004
0.000389	0.000252	0.000355	0.000248	0.000359	0.00044	0.000312	0.000317	0.00033	0.000485	0.000892	0.000278	0.000186	0.000259	0.000374
0.000218	0.000155	0.000192	0.000152	0.000202	0.000261	0.000184	0.000194	0.000183	0.000279	0.000278	0.000526	0.000118	0.000152	0.000217
0.000148	0.0001	0.000134	0.000102	0.00014	0.000175	0.000124	0.000129	0.000125	0.000191	0.000186	0.000118	0.000175	0.000103	0.000146
0.000215	0.000135	0.000201	0.000146	0.000211	0.000251	0.000179	0.000183	0.00018	0.00028	0.000259	0.000152	0.000103	0.000226	0.00021
0.000346	0.000218	0.000316	0.000195	0.000298	0.000365	0.000264	0.000255	0.000292	0.0004	0.000374	0.000217	0.000146	0.00021	0.000468

Table 36: Expected Daily Return and Optimal Weight for each stock

	Portfolio Weights	Expected Return
BAC	-6.9234	-0.0007
BP	-0.6435	-0.0003
C	2.5552	0.0019
EMC	3.4468	0.0015
F	1.8904	0.002
FCX	3.2035	0.0027
GE	0.1176	0.0009
GLW	-1.1427	0.0008
JPM	-2.1581	0.0003
LVS	4.1641	0.0054
LYG	0.3723	0.0016
NOK	-2.6583	-0.0006
PFE	-0.662	0.0003
TWX	-0.7045	0.0008
WFC	0.1425	0.0009

Table 37: The extended Fama & French Model with both Corporate Premium factor and International Premium factor (EFFMCI)

	β0	β1	β2	β3	β4	β5	R-squared	F-value	p-value	error variance
BAC	-0.00464	0.01097	-0.006	0.017736	0.001061	-0.00334	0.701975	97.98539	1.01E-52	0.000159
BP	0.04413	0.009582	-0.00901	0.007014	-0.00773	-0.0153	0.191324	9.842118	1.85E-08	0.000675
C	-0.00344	0.008803	-0.00112	0.01748	0.000747	0.002852	0.538674	48.57475	3.71E-33	0.000272
EMC	-0.01664	0.011147	0.001178	-0.00483	0.003015	0.004808	0.562576	53.50214	1.56E-35	0.000117
F	0.012552	0.011675	0.00422	0.005891	-0.00287	0.004946	0.549758	50.79469	3.04E-34	0.000254
FCX	0.069612	0.016307	-0.00203	0.005552	-0.01312	-0.0093	0.636456	72.82893	8.26E-44	0.000256
GE	-0.00318	0.011476	-0.00176	0.004154	0.00123	-0.00598	0.726727	110.6287	1.29E-56	8.43E-05
GLW	-0.00973	0.013787	-0.00132	-0.00299	0.001693	0.002104	0.545724	49.97429	7.61E-34	0.000193
JPM	0.017727	0.008901	-0.00609	0.016821	-0.00372	0.001199	0.738871	117.7084	1.17E-58	9.61E-05
LVS	0.006636	0.016956	0.001685	0.006294	-0.00159	0.010941	0.466221	36.33489	1.16E-26	0.000619
LYG	-0.05315	0.018668	-0.00645	0.002365	0.006673	0.045311	0.530514	47.00752	2.24E-32	0.000429
NOK	0.039234	0.015059	-0.00893	-0.00716	-0.00743	-0.0103	0.362401	23.64474	8.56E-19	0.000344
PFE	0.010361	0.008826	-0.00357	-0.00252	-0.00251	0.003145	0.439298	32.59276	1.78E-24	0.000101
TWX	-0.02589	0.009985	0.000748	0.000226	0.004264	0.010648	0.67005	84.47967	3.72E-48	7.62E-05
WFC	0.003306	0.012069	-0.00585	0.012701	0.000896	-0.01579	0.708578	101.1481	9.96E-54	0.00014

Table 38: Covariance Matrix

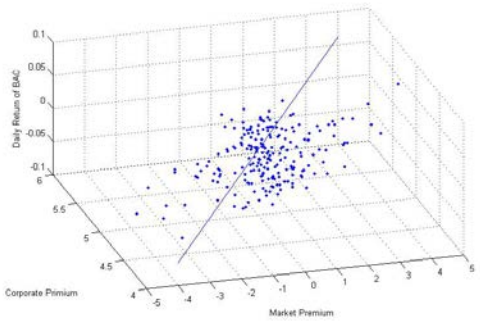
0.000521	0.000227	0.000334	0.000197	0.000308	0.000375	0.000272	0.000259	0.00031	0.000414	0.000389	0.000218	0.000148	0.000216	0.000346
0.000227	0.000816	0.000201	0.000124	0.000188	0.000245	0.000171	0.000166	0.000196	0.000257	0.000248	0.000159	0.000102	0.000132	0.000218
0.000334	0.000201	0.000576	0.000184	0.000292	0.000346	0.000251	0.000239	0.000282	0.000387	0.000356	0.000192	0.000134	0.000201	0.000316
0.000197	0.000124	0.000184	0.00026	0.000203	0.000239	0.000172	0.00018	0.000162	0.000269	0.000249	0.000151	0.000101	0.000147	0.000195
0.000308	0.000188	0.000292	0.000203	0.000552	0.000356	0.000253	0.000254	0.000258	0.000398	0.000358	0.000204	0.000141	0.00021	0.000298
0.000375	0.000245	0.000346	0.000239	0.000356	0.000689	0.000304	0.000306	0.000319	0.000474	0.000434	0.000268	0.000178	0.000247	0.000364
0.000272	0.000171	0.000251	0.000172	0.000253	0.000304	0.000301	0.000219	0.000228	0.000337	0.000312	0.000184	0.000124	0.000179	0.000264
0.000259	0.000166	0.000239	0.00018	0.000254	0.000306	0.000219	0.000414	0.000215	0.000339	0.000318	0.000194	0.000129	0.000183	0.000255
0.00031	0.000196	0.000282	0.000162	0.000258	0.000319	0.000228	0.000215	0.000359	0.000347	0.000328	0.000185	0.000126	0.000179	0.000292
0.000414	0.000257	0.000387	0.000269	0.000398	0.000474	0.000337	0.000339	0.000347	0.001132	0.000484	0.00028	0.000191	0.00028	0.0004
0.000389	0.000248	0.000356	0.000249	0.000358	0.000434	0.000312	0.000318	0.000328	0.000484	0.000892	0.000275	0.000185	0.000261	0.000375
0.000218	0.000159	0.000192	0.000151	0.000204	0.000268	0.000184	0.000194	0.000185	0.00028	0.000275	0.000526	0.000119	0.00015	0.000217
0.000148	0.000102	0.000134	0.000101	0.000141	0.000178	0.000124	0.000129	0.000126	0.000191	0.000185	0.000119	0.000175	0.000102	0.000146
0.000216	0.000132	0.000201	0.000147	0.00021	0.000247	0.000179	0.000183	0.000179	0.00028	0.000261	0.00015	0.000102	0.000226	0.00021
0.000346	0.000218	0.000316	0.000195	0.000298	0.000364	0.000264	0.000255	0.000292	0.0004	0.000375	0.000217	0.000146	0.00021	0.000468

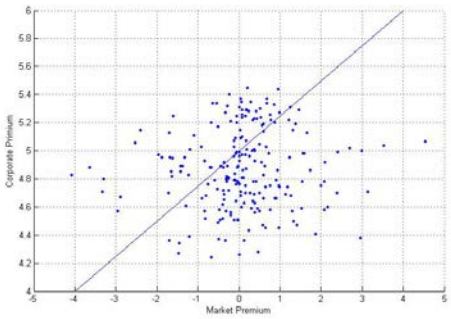
Table 39: Expected Daily Return and Optimal Weight for each stock

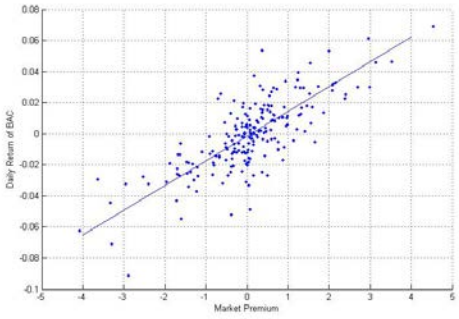
	Portfolio Weights	Expected Return
BAC	-6.8902	-0.0007
BP	-0.6545	-0.0003
C	2.5433	0.0019
EMC	3.4713	0.0015
F	1.859	0.002
FCX	3.2653	0.0027
GE	0.1217	0.0009
GLW	-1.1382	0.0008
JPM	-2.2076	0.0003
LVS	4.1334	0.0054
LYG	0.3833	0.0016
NOK	-2.7007	-0.0006
PFE	-0.6735	0.0003
TWX	-0.6561	0.0008
WFC	0.1436	0.0009

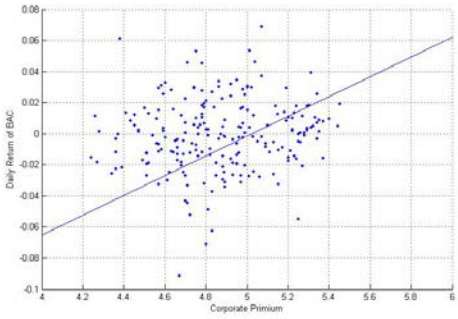
Regression Plot Example:

Take the CAPMC model for example. We regress the daily returns of BAC on the Market Premium factor and Corporate Premium factor and then create the 3-dimensional plots as shown below. The specific R-squared value of this regression is 0.598646, and from the plots we can see that the daily stock return of BAC has a relatively strong relationship with the market premium. Also the variance of the factor coefficients of corporate premium and international premium seems large due to the relative dispersal of the data points.









Comparison:

I. Comparison among factors

EFFMCI contains all of our 5 factors. To see if the performances of the five factors have reliable relationships with each stock in our portfolio, we computed the p-values of each β under each stock.

Table 40: The result of 5 factors in the EFFMCI model

p-value	β0	β1	β2	β3	β4	β5
BAC	0.794356	9.15E-19	0.000611	3.18E-12	0.744846	0.751707
BP	0.233525	5.15E-05	0.011922	0.156789	0.252511	0.478006
C	0.884035	9.26E-09	0.618574	7.43E-08	0.861877	0.835759
EMC	0.278565	2.95E-24	0.42584	0.019372	0.282028	0.589389
F	0.580335	2.28E-14	0.054248	0.053118	0.488896	0.708984
FCX	0.002564	9.2E-24	0.355654	0.06942	0.001841	0.487583
GE	0.809639	6.98E-32	0.161516	0.018084	0.606882	0.431072
GLW	0.621832	6.4E-23	0.488358	0.257721	0.638014	0.854166
JPM	0.206099	5.08E-20	9.5E-06	1.15E-16	0.14541	0.879452
LVS	0.851954	7.61E-13	0.620844	0.184363	0.805458	0.595791
LYG	0.073063	8.22E-20	0.023701	0.548421	0.216433	0.00907
NOK	0.138528	6.88E-17	0.000525	0.043306	0.123707	0.504904
PFE	0.468401	4.45E-19	0.01001	0.187409	0.335767	0.706086
TWX	0.038225	3.93E-28	0.531665	0.892033	0.061061	0.142719
WFC	0.844369	7.01E-24	0.000364	4.97E-08	0.769915	0.108572

Here we consider a 0.95 confidence level. If the p-value is larger than 0.05, we cannot reject the hypothesis that the factor is not zero, which means the factor may be meaningless. We can see most of the p-values for β4 (the corporate premium) and β5 (the international premium) are too large, which means they statistically have little influence on the stocks' return. For β1 (the market premium), all p-values are significantly small. We thus can conclude that the market premium is a strong factor that can significantly explain part of the stocks' risk.

When considering the intercept β0, we can see most of the p-values are large. A belief of a zero intercept indicates there is little mispricing, so we can say only FCX and TWX are mispriced with high probability.

II. Comparison among models

First, from the F-values and p-values we can say that with 95% confidence all of our models are meaningful and those factors together in each model can be well used to explain the stock returns, since p-values are all smaller than 0.05 (actually they are far smaller than 0.05 to which extent we can even ignore them). We can see most of the p-values for β4 (the corporate premium) and β5 (the international premium) are too large, which means they statistically have little influence on the stocks' return. The reasons why these two factors have little influence on the stock's return are understandable. First, Moody's AAA bond yield chooses bonds of which the remaining maturities are more than 20 years. We know that bonds have many types of maturities, so these bonds are only a small part of the entire bond pool. In addition, since the holding period of our portfolio is 10 months, the long-term bond yield may have little effect on the portfolio's return. This factor does not take into account bonds which have less credit, which are all reflections of the economy. Second, although many of the companies have headquarters outside of the USA (eg, NOK), the business activities in the USA are still very important to those companies. The international premium might have an effect, but it is not determinate. We examine Table 41 which shows the F-values of the CAPMC and EFFMC.

Table 41: The F-values of The CAPMC and EFFMC

F-value	EFFMC	CAPMC
BAC	122.9847	157.3603
BP	12.2059	19.3444
C	60.9867	92.5284
EMC	67.034	128.0718
F	63.7207	121.2488
FCX	91.1359	178.0334
GE	138.3853	265.0358
GLW	62.7497	124.8434
JPM	147.8219	158.1009
LVS	45.505	90.1285
LYG	55.4409	104.7942
NOK	29.5217	48.5027
PFE	40.8729	75.3535
TWX	104.4798	210.6178
WFC	124.8338	185.5634

We compare the R-squared value between the Fama and French model class as shown below. Since the larger the R-squared value the better the explanation power of the model, we can see that the corporate premium has a little more power than the international premium in explaining our stock returns. The EFFMCI model is the best of the four.

Table 42: R-squared value between the Fama and French model class

	FFM	EFFMC	EFFMI	EFFMCI
BAC	0.701562	0.701829	0.701824	0.701975
BP	0.185575	0.189368	0.186211	0.191324
C	0.538545	0.538577	0.538606	0.538674
EMC	0.560012	0.56197	0.560139	0.562576
F	0.547961	0.549455	0.548718	0.549758
FCX	0.618964	0.635599	0.619028	0.636456
GE	0.725143	0.725916	0.726376	0.726727
GLW	0.545234	0.545651	0.545241	0.545724
JPM	0.735759	0.738844	0.736182	0.738871
LVS	0.465065	0.4655	0.466065	0.466221
LYG	0.514193	0.514815	0.527029	0.530514
NOK	0.354882	0.361026	0.355068	0.362401
PFE	0.435564	0.438914	0.436791	0.439298
TWX	0.662899	0.666624	0.664427	0.67005
WFC	0.704101	0.704942	0.708458	0.708578

We compare the R-squared value between the CAPM class as shown below. Again the corporate spread works better than international spread.

Table 43: The R-squared between the CAPM class

	CAPMC	CAPMI	CAPMCI
BAC	0.598646	0.595489	0.598651
BP	0.154948	0.151862	0.155688
C	0.467248	0.464691	0.467401
EMC	0.548319	0.547671	0.548751
F	0.534727	0.534559	0.534785
FCX	0.62791	0.613628	0.628546
GE	0.715277	0.714772	0.71581
GLW	0.541988	0.54198	0.542136
JPM	0.599774	0.600284	0.600293
LVS	0.460712	0.4613	0.4613
LYG	0.498322	0.51351	0.517334
NOK	0.314947	0.302854	0.3152
PFE	0.416655	0.413489	0.417847
TWX	0.666264	0.663382	0.669416
WFC	0.637536	0.637771	0.639639

Then we examine the relative performance between the two kinds of model classes. As a whole the Fama and French class does better than the CAPM class, especially for BAC and JPM, both of which are in the financial industry. Because of such evidence, we can make a hypothesis that SMB and HML might have a deeper effect in the financial industry.

4. PORTFOLIO PERFORMANCE ANALYSIS:

After generating the optimal weights, we are ready to form our simulative optimal portfolios. Now assume we have $500,000 initial capital and try to form a portfolio which exactly has a total value of $500,000 (here we assume that we can hold non-integer shares of stock, e.g. 500.23 shares of BAC). We here use the closing prices of Jan 3rd, 2011 as our purchase prices and use the closing prices of Oct 31, 2011 to determine our end-value.

To see if the market behaves as our models expect, we compounded the expected daily return (calculated as the average daily return from Mar 1st, 2010 to Dec 31st, 2010) 210 times (the number of market days from Jan 3rd, 2011 to Oct 31st, 2011) and multiply it by the initial purchase price to obtain the expected end stock value. Thus we can compare the objective portfolio return and actual portfolio return with a 10-month holding period.

Data are shown in the following tables.

Table 44 : The Fama & French Model

	Weights	1/3/2011	Position	Expected 10/31	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3460950	14.15	-244590	12.21498	-2987664	6.83	-1670550	-0.13675	-0.51731
BP	-326700	43.47	-7515.53	40.81549	-306750	43.75	-328804	-0.06107	0.006441
C	1278450	48.96	26112.13	72.93912	1904596	31.58	824621.1	0.48977	-0.35498
EMC	1717300	23.11	74309.82	31.65922	2352591	24.51	1821334	0.369936	0.06058
F	952100	17.25	55194.2	26.24283	1448452	11.68	644668.3	0.521323	-0.3229
FCX	1599450	58.58	27303.69	103.1959	2817629	40.26	1099246	0.761624	-0.31273
GE	12450	17.84	697.87	21.54962	15038.83	16.71	11661.41	0.207938	-0.06334
GLW	-573700	18.94	-30290.4	22.40331	-678605	14.22	-430729	0.182857	-0.24921
JPM	-1050300	42.66	-24620.3	45.43362	-1118587	34.76	-855800	0.065017	-0.18519
LVS	2081300	45.59	45652.56	141.265	6449108	46.95	2143387	2.098596	0.029831
LYG	221050	4.17	53009.59	5.833677	309240.8	2.06	109199.8	0.398963	-0.506
NOK	-1333250	10.04	-132794	8.851078	-1175369	6.73	-893702	-0.11842	-0.32968
PFE	-312900	16.97	-18438.4	18.07333	-333244	19.07	-351621	0.065017	0.123748
TWX	-319700	31.72	-10078.8	37.52023	-378159	34.99	-352658	0.182857	0.10309
WFC	15350	31.11	493.4105	37.57896	18541.85	25.78	12720.12	0.207938	-0.17133
Total	500000				8336820		1782973		

Table 45: The Two-Factor Model with Corporate Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3069850	14.15	-216951	12.21498	-2650047	6.83	-1481772	-0.13675	-0.51731
BP	-477850	43.47	-10992.6	40.81549	-448670	43.75	-480928	-0.06107	0.006441
C	926950	48.96	18932.8	72.93912	1380942	31.58	597897.9	0.48977	-0.35498
EMC	2208500	23.11	95564.69	31.65922	3025504	24.51	2342291	0.369936	0.06058
F	1150350	17.25	66686.96	26.24283	1750054	11.68	778903.7	0.521323	-0.3229
FCX	1575050	58.58	26887.16	103.1959	2774645	40.26	1082477	0.761624	-0.31273
GE	-24400	17.84	-1367.71	21.54962	-29473.7	16.71	-22854.5	0.207938	-0.06334
GLW	-397600	18.94	-20992.6	22.40331	-470304	14.22	-298515	0.182857	-0.24921
JPM	-1493050	42.66	-34998.8	45.43362	-1590123	34.76	-1216559	0.065017	-0.18519
LVS	2107850	45.59	46234.92	141.265	6531376	46.95	2170729	2.098596	0.029831
LYG	99800	4.17	23932.85	5.833677	139616.5	2.06	49301.68	0.398963	-0.506
NOK	-1385500	10.04	-137998	8.851078	-1221431	6.73	-928727	-0.11842	-0.32968
PFE	-426650	16.97	-25141.4	18.07333	-454389	19.07	-479447	0.065017	0.123748
TWX	179050	31.72	5644.704	37.52023	211790.6	34.99	197508.2	0.182857	0.10309
WFC	-472650	31.11	-15192.9	37.57896	-570932	25.78	-391672	0.207938	-0.17133
Total	500000				8378558		1918634		

Table 46: The extended Fama & French Model with Corporate Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3442500	14.15	-243286	12.21498	-2971737	6.83	-1661645	-0.13675	-0.51731
BP	-334300	43.47	-7690.36	40.81549	-313886	43.75	-336453	-0.06107	0.006441
C	1274750	48.96	26036.56	72.93912	1899084	31.58	822234.6	0.48977	-0.35498
EMC	1736500	23.11	75140.63	31.65922	2378894	24.51	1841697	0.369936	0.06058
F	933050	17.25	54089.86	26.24283	1419471	11.68	631769.5	0.521323	-0.3229
FCX	1613550	58.58	27544.38	103.1959	2842468	40.26	1108937	0.761624	-0.31273
GE	28700	17.84	1608.744	21.54962	34667.83	16.71	26882.12	0.207938	-0.06334
GLW	-568150	18.94	-29997.4	22.40331	-672040	14.22	-426562	0.182857	-0.24921
JPM	-1091000	42.66	-25574.3	45.43362	-1161933	34.76	-888963	0.065017	-0.18519
LVS	2067600	45.59	45352.05	141.265	6406658	46.95	2129279	2.098596	0.029831
LYG	224200	4.17	53764.99	5.833677	313647.6	2.06	110755.9	0.398963	-0.506
NOK	-1358300	10.04	-135289	8.851078	-1197452	6.73	-910494	-0.11842	-0.32968
PFE	-331800	16.97	-19552.2	18.07333	-353373	19.07	-372860	0.065017	0.123748
TWX	-283100	31.72	-8924.97	37.52023	-334867	34.99	-312285	0.182857	0.10309
WFC	30750	31.11	988.4282	37.57896	37144.1	25.78	25481.68	0.207938	-0.17133
Total	500000				8326746		1787775		

Table 47: The extended CAPM Model with International Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3071350	14.15	-217057	12.21498	-2651341	6.83	-1482496	-0.13675	-0.51731
BP	-468900	43.47	-10786.7	40.81549	-440266	43.75	-471920	-0.06107	0.006441
C	920700	48.96	18805.15	72.93912	1371631	31.58	593866.5	0.48977	-0.35498
EMC	2206150	23.11	95463	31.65922	3022284	24.51	2339798	0.369936	0.06058
F	1165450	17.25	67562.32	26.24283	1773026	11.68	789127.9	0.521323	-0.3229
FCX	1571900	58.58	26833.39	103.1959	2769096	40.26	1080312	0.761624	-0.31273
GE	-13850	17.84	-776.345	21.54962	-16729.9	16.71	-12972.7	0.207938	-0.06334
GLW	-399750	18.94	-21106.1	22.40331	-472847	14.22	-300129	0.182857	-0.24921
JPM	-1502050	42.66	-35209.8	45.43362	-1599708	34.76	-1223893	0.065017	-0.18519
LVS	2124350	45.59	46596.84	141.265	6582503	46.95	2187722	2.098596	0.029831
LYG	74150	4.17	17781.77	5.833677	103733.1	2.06	36630.46	0.398963	-0.506
NOK	-1347450	10.04	-134208	8.851078	-1187887	6.73	-903221	-0.11842	-0.32968
PFE	-417050	16.97	-24575.7	18.07333	-444165	19.07	-468659	0.065017	0.123748
TWX	121500	31.72	3830.391	37.52023	143717.1	34.99	134025.4	0.182857	0.10309
WFC	-463800	31.11	-14908.4	37.57896	-560242	25.78	-384338	0.207938	-0.17133
Total	500000				8392803		1913853		

Table 48: The extended CAPM Model with both Corporate Premium factor and International Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3068400	14.15	-216848	12.214981	-2648795	6.83	-1481072	-0.13675	-0.51731
BP	-474550	43.47	-10916.7	40.815487	-445571	43.75	-477607	-0.06107	0.006441
C	926450	48.96	18922.59	72.9391161	1380197	31.58	597575.4	0.48977	-0.35498
EMC	2208750	23.11	95575.51	31.6592198	3025846	24.51	2342556	0.369936	0.06058
F	1150450	17.25	66692.75	26.2428272	1750206	11.68	778971.4	0.521323	-0.3229
FCX	1585400	58.58	27063.84	103.195915	2792878	40.26	1089590	0.761624	-0.31273
GE	-5850	17.84	-327.915	21.5496188	-7066.44	16.71	-5479.46	0.207938	-0.06334
GLW	-398400	18.94	-21034.8	22.4033147	-471250	14.22	-299116	0.182857	-0.24921
JPM	-1499250	42.66	-35144.2	45.4336157	-1596726	34.76	-1221611	0.065017	-0.18519
LVS	2107250	45.59	46221.76	141.265002	6529517	46.95	2170112	2.098596	0.029831
LYG	79800	4.17	19136.69	5.8336771	111637.3	2.06	39421.58	0.398963	-0.506
NOK	-1382100	10.04	-137659	8.85107835	-1218434	6.73	-926448	-0.11842	-0.32968
PFE	-432450	16.97	-25483.2	18.0733347	-460567	19.07	-485965	0.065017	0.123748
TWX	160150	31.72	5048.865	37.5202292	189434.6	34.99	176659.8	0.182857	0.10309
WFC	-457250	31.11	-14697.8	37.5789597	-552330	25.78	-378910	0.207938	-0.17133
Total	500000				8378977		1918678		

Table 49: The extended Fama & French Model with International Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3461700	14.15	-244643	12.21498	-2988311	6.83	-1670912	-0.13675	-0.51731
BP	-321750	43.47	-7401.66	40.81549	-302102	43.75	-323822	-0.06107	0.006441
C	1277600	48.96	26094.77	72.93912	1903330	31.58	824072.9	0.48977	-0.35498
EMC	1723400	23.11	74573.78	31.65922	2360948	24.51	1827803	0.369936	0.06058
F	945200	17.25	54794.2	26.24283	1437955	11.68	639996.3	0.521323	-0.3229
FCX	1601750	58.58	27342.95	103.1959	2821681	40.26	1100827	0.761624	-0.31273
GE	58800	17.84	3295.964	21.54962	71026.77	16.71	55075.56	0.207938	-0.06334
GLW	-571350	18.94	-30166.3	22.40331	-675825	14.22	-428965	0.182857	-0.24921
JPM	-1079050	42.66	-25294.2	45.43362	-1149206	34.76	-879226	0.065017	-0.18519
LVS	2082050	45.59	45669.01	141.265	6451432	46.95	2144160	2.098596	0.029831
LYG	186150	4.17	44640.29	5.833677	260417	2.06	91958.99	0.398963	-0.506
NOK	-1329150	10.04	-132385	8.851078	-1171754	6.73	-890954	-0.11842	-0.32968
PFE	-331000	16.97	-19505	18.07333	-352521	19.07	-371961	0.065017	0.123748
TWX	-352250	31.72	-11105	37.52023	-416661	34.99	-388563	0.182857	0.10309
WFC	71250	31.11	2290.26	37.57896	86065.6	25.78	59042.91	0.207938	-0.17133
Total	500000				8336473		1788533		

Table 50: The extended Fama & French Model with both Corporate Premium factor and International Premium factor

	Weights	1/3/2011	Position	Expected 10/31/2011	Expected Value	10/31/2011	Value	Expected Return	Actual Return
BAC	-3445100	14.15	-243470	12.21498	-2973981	6.83	-1662900	-0.13675	-0.51731
BP	-327250	43.47	-7528.18	40.81549	-307266	43.75	-329358	-0.06107	0.006441
C	1271650	48.96	25973.24	72.93912	1894465	31.58	820235	0.48977	-0.35498
EMC	1735650	23.11	75103.85	31.65922	2377729	24.51	1840795	0.369936	0.06058
F	929500	17.25	53884.06	26.24283	1414070	11.68	629365.8	0.521323	-0.3229
FCX	1632650	58.58	27870.43	103.1959	2876115	40.26	1122064	0.761624	-0.31273
GE	60850	17.84	3410.874	21.54962	73503.04	16.71	56995.71	0.207938	-0.06334
GLW	-569100	18.94	-30047.5	22.40331	-673164	14.22	-427276	0.182857	-0.24921
JPM	-1103800	42.66	-25874.4	45.43362	-1175566	34.76	-899393	0.065017	-0.18519
LVS	2066700	45.59	45332.31	141.265	6403869	46.95	2128352	2.098596	0.029831
LYG	191650	4.17	45959.23	5.833677	268111.3	2.06	94676.02	0.398963	-0.506
NOK	-1350350	10.04	-134497	8.851078	-1190444	6.73	-905165	-0.11842	-0.32968
PFE	-336750	16.97	-19843.8	18.07333	-358644	19.07	-378422	0.065017	0.123748
TWX	-328050	31.72	-10342.1	37.52023	-388036	34.99	-361869	0.182857	0.10309
WFC	71800	31.11	2307.94	37.57896	86729.97	25.78	59498.68	0.207938	-0.17133
Total	500000				8327492		1787601		

The **actual covariance matrix** for the holding period of Jan 3rd, 2011 to Oct 31st, 2011 is shown in Table 52.

Table 51 : The actual covariance matrix for the holding period of Jan 3rd, 2011 to Oct 31st, 2011

BAC	BP	C	EMC	F	FCX	GE	GLW	JPM	LVS	LYG	NOK	PFE	TWX	WFC
0.001379	0.000385	0.001072	0.000409	0.000584	0.000628	0.000469	0.000577	0.00076	0.000553	0.00084	0.000627	0.000331	0.000411	0.000717
0.000385	0.000383	0.000395	0.000243	0.0003	0.000358	0.000257	0.000305	0.000303	0.000308	0.000435	0.00034	0.000185	0.000222	0.000289
0.001072	0.000395	0.001046	0.000388	0.000562	0.000609	0.000457	0.000546	0.000698	0.000513	0.000818	0.000588	0.000304	0.000396	0.000662
0.000409	0.000243	0.000388	0.000367	0.00033	0.000357	0.000261	0.000352	0.000296	0.000342	0.000446	0.000302	0.00018	0.00025	0.000282
0.000584	0.0003	0.000562	0.00033	0.000634	0.000466	0.000313	0.000398	0.000425	0.000419	0.000546	0.000367	0.00023	0.000302	0.000417
0.000628	0.000358	0.000609	0.000357	0.000466	0.000867	0.000335	0.000483	0.000475	0.000524	0.000595	0.000438	0.000268	0.000349	0.00046
0.000469	0.000257	0.000457	0.000261	0.000313	0.000335	0.00036	0.000333	0.000356	0.000335	0.000503	0.000377	0.000204	0.000254	0.000351
0.000577	0.000305	0.000546	0.000352	0.000398	0.000483	0.000333	0.000663	0.000409	0.000387	0.000577	0.000483	0.000275	0.000293	0.000396
0.00076	0.000303	0.000698	0.000296	0.000425	0.000475	0.000356	0.000409	0.000583	0.00038	0.000648	0.000443	0.000238	0.000311	0.000514
0.000553	0.000308	0.000513	0.000342	0.000419	0.000524	0.000335	0.000387	0.00038	0.00088	0.000519	0.00033	0.000243	0.000317	0.000371
0.00084	0.000435	0.000818	0.000446	0.000546	0.000595	0.000503	0.000577	0.000648	0.000519	0.001316	0.000693	0.000337	0.000425	0.0006
0.000627	0.00034	0.000588	0.000302	0.000367	0.000438	0.000377	0.000483	0.000443	0.00033	0.000693	0.001131	0.000243	0.00032	0.000421
0.000331	0.000185	0.000304	0.00018	0.00023	0.000268	0.000204	0.000275	0.000238	0.000243	0.000337	0.000243	0.000235	0.000174	0.000246
0.000411	0.000222	0.000396	0.00025	0.000302	0.000349	0.000254	0.000293	0.000311	0.000317	0.000425	0.00032	0.000174	0.000352	0.000299
0.000717	0.000289	0.000662	0.000282	0.000417	0.00046	0.000351	0.000396	0.000514	0.000371	0.0006	0.000421	0.000246	0.000299	0.000586

Comparison:

First, we notice that we compounded the expected daily return 210 times to get the expected 10-month return, and thus on each stock we would expect to receive a large return no matter long or short. Furthermore, we did not limit our ability to short, which means we can form any portfolio we want. Both of these prerequisites tend to stretch out our expected return by a substantial amount (e.g. about 209.9% expected return for LVS).

Consider the portfolio returns shown below. We notice that the portfolios in the CAPM class are expected to generate a little higher return than those in the Fama and French class, and actually they do. Then we see in more depth that within the Fama and French class, the EFFMI portfolio has the largest actual return 257.7066%, and CAPMCI, which has a return of 283.7356%, stands out of the CAPM class. Among all the portfolios, the CAPMCI model performs the best. The original Fama and French model generates the smallest return, which means that the two extended factors we add do have some positive effect.

Table 52: Portfolio return

	Initial Value	Expected Value	Expected Return	Actual Value	Actual Return
FFM	500000	8336820	15.67364	1782973	2.565947
EFFMC	500000	8326746	15.65349	1787775	2.575549
EFFMI	500000	8336473	15.67295	1788533	2.577066
EFFMCI	500000	8327492	15.65498	1787601	2.575201
CAPMC	500000	8378558	15.75712	1918634	2.837269
CAPMI	500000	8392803	15.78561	1913853	2.827707
CAPMCI	500000	8378977	15.75795	1918678	2.837356

5. CONCLUSION:

While the CAPM is a single-factor model with the market index being the factor, multiple-factor models are more realistic and are widely used in practice. Based on the back-testing method we use and the results we generate, we can make some conclusions.

First, the large F-value and small p-value indicate that as a whole the factors we use are meaningful. We believe that they all have linear relationships with the stock returns and thus can be used to form credible models and generate positive expected returns.

Second, as we see from the p-values, the risk explanation power of the corporate premium factor and international premium factor are low. The SML and HML factor are ‘strong’ over some stocks and ‘weak’ over the others. The market premium can well-explain the risks, and the relative high p-values of the intercept term means there is little mispricing.

Among all the factor models the CAPMC performs best under our back-testing method. If we had performed this analysis prior to initially forming our optimal portfolio in the prior project, we would like to do the back-testing first and select a more highly ‘profitable’ stock combination.

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